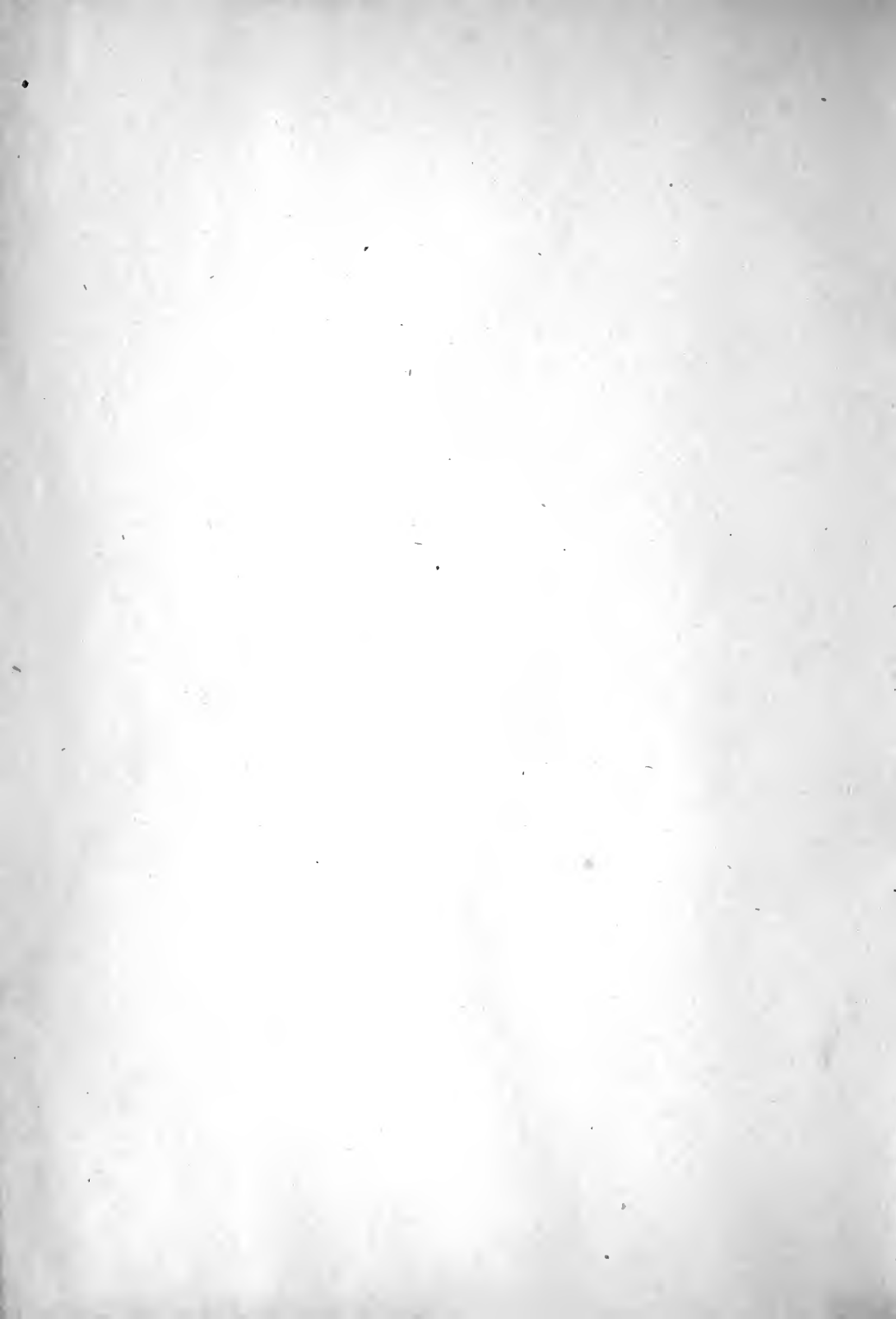


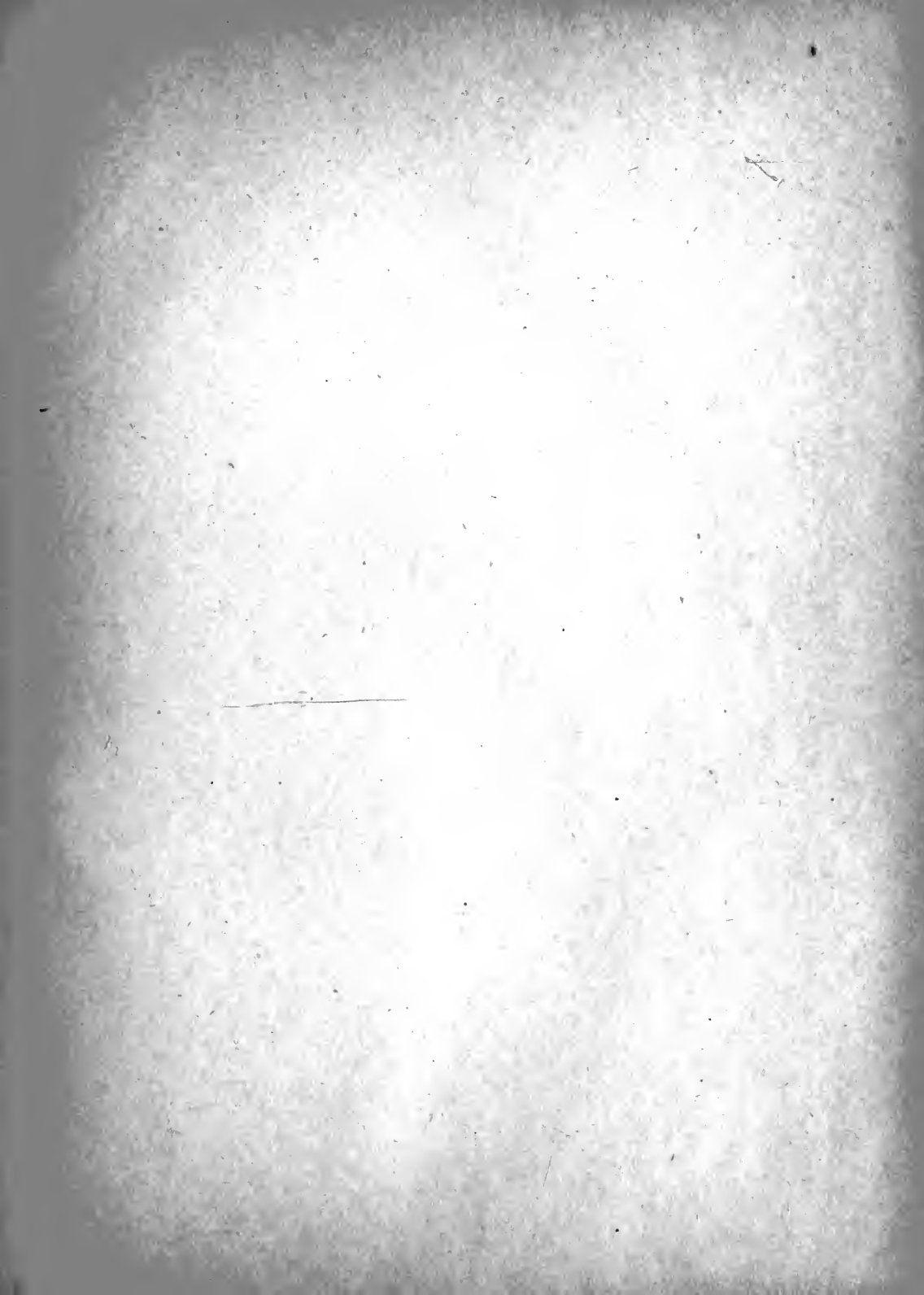
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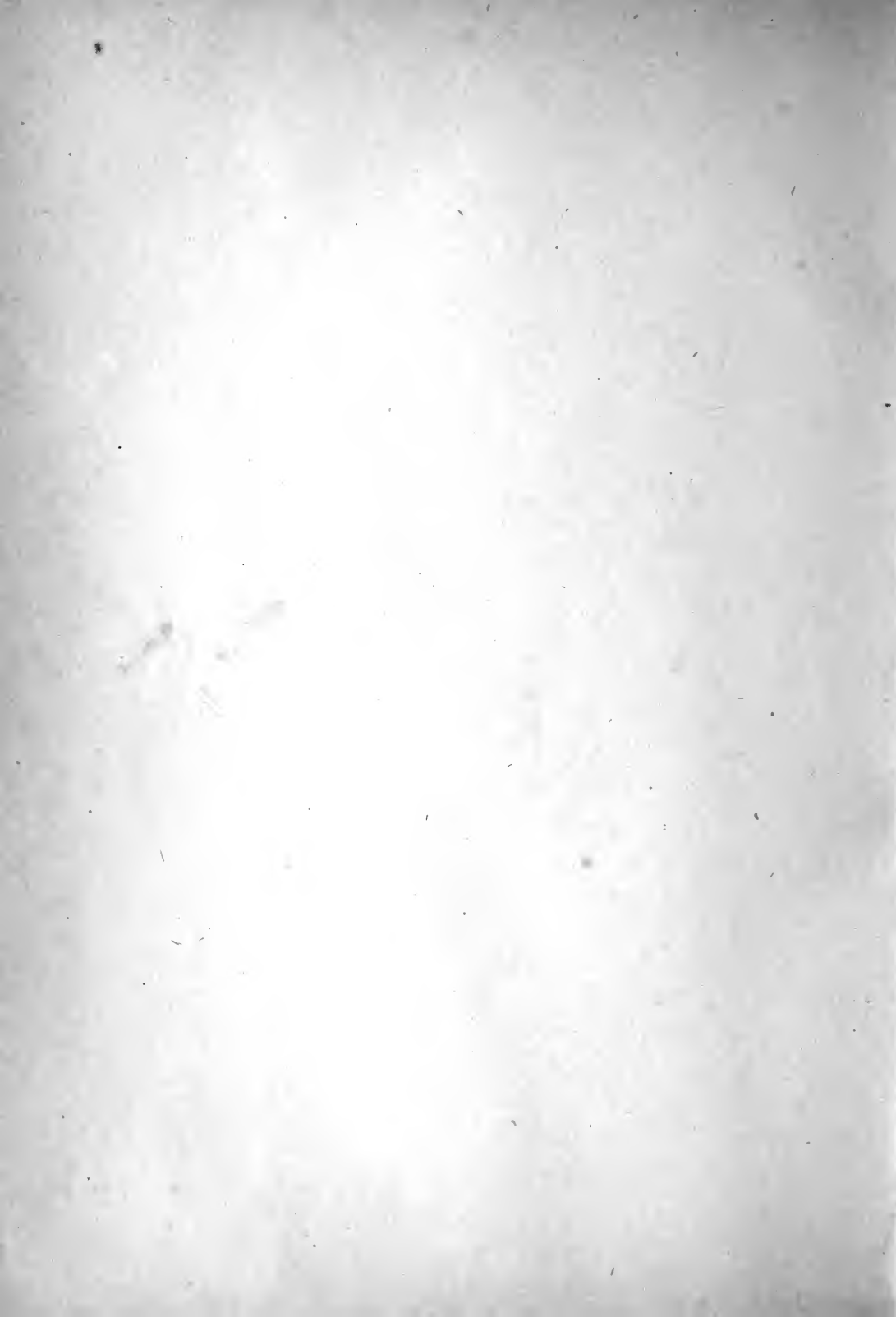
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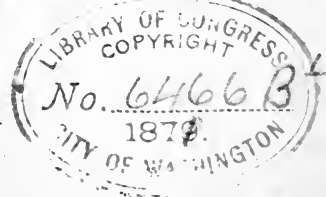
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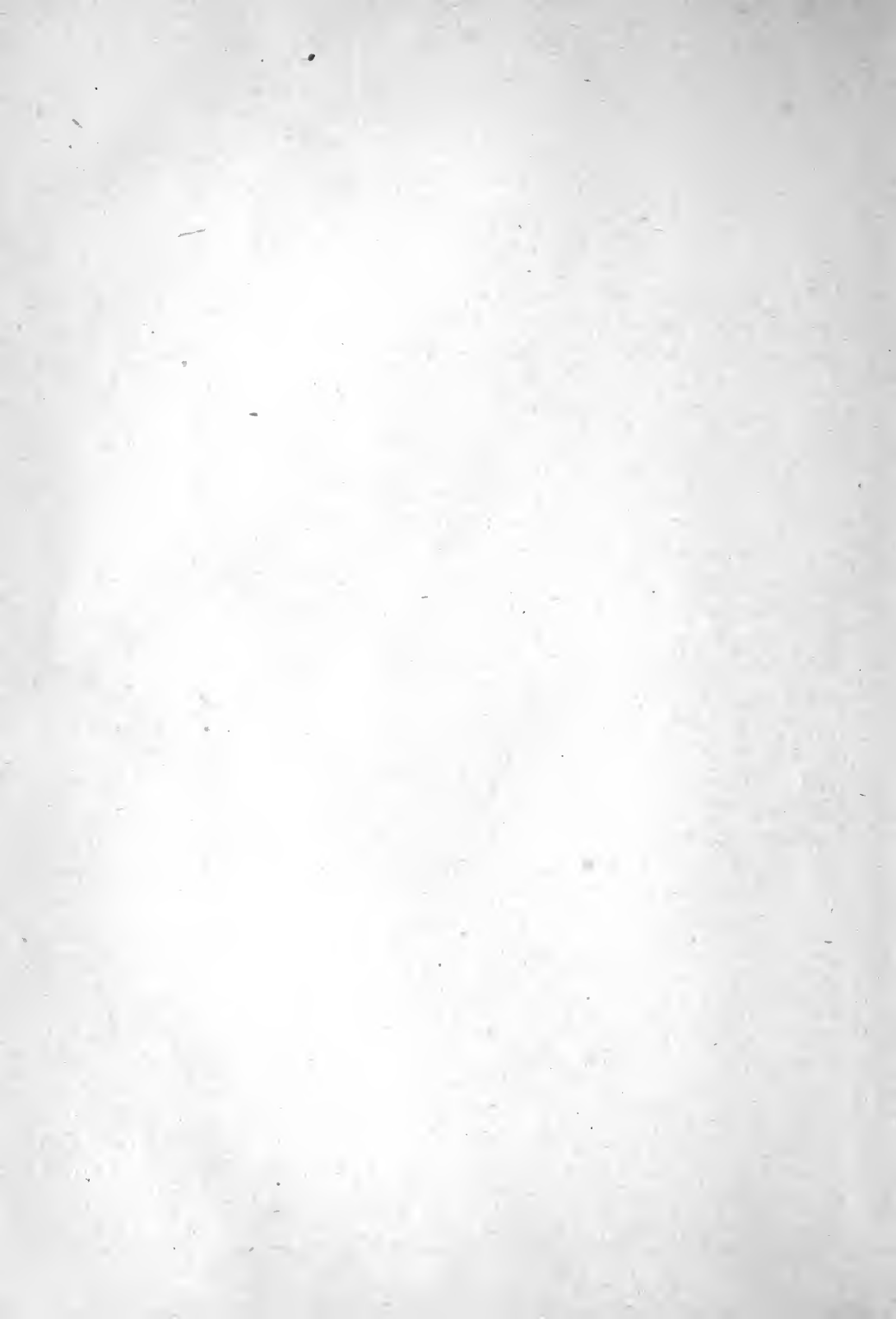
P R E F A C E .

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In offering a new system of philosophy to the scientific world, the author is aware that many will say of it at the outset as Omer Pasha did of the Alexandrian Library, "If it contain the Koran, we have it already, and whatever else it may contain is not worth having." We can only remind such persons that the present age is one of free inquiry, that the human mind at best is very feeble and easily deceived by appearances, and that though we may be contented and confirmed in our opinions, which are supported by the names of distinguished philosophers; yet our condition, perhaps, may be no more happy than that of Pollock's rustic, who was confirmed in the belief that the visual line which girt him round about was the world's extreme. Fortunately for the human race, however, there is a class of men in America and Europe, whose reflections teach them, that there can be no great advance in civilization without an increase of knowledge in science. To such persons the author must look for a fair examination of the present work. And in calling the attention of seekers after scientific truth in a preface, little more can be done by an author than to promise that, in the author's opinion, a thorough perusal will repay the reader.

The thoughts contained in this book have been gained by much labor, and have not been set down without much reflection. The subject is a profound one; and it is, indeed, to the matured philosopher, whose mind has been grappling with intricate scientific questions, and who can command and concentrate his thoughts, that, in the first instance, the value of new scientific truths must be perceived and appreciated. To such matured minds we say, read our book carefully, and speak your minds freely respecting its merits. For we believe, that the philosophic seeker for truth, of the present and future ages, will find in it sufficient immutable and valuable truth to approve and justify the time and labor spent by the author in his attempt to correct errors and to open the gates of scientific truth to mankind. THE AUTHOR.

EUREKA, California, January 25, 1879.



INTRODUCTION.

“Philosophy and the intellectual sciences like statues,” says Bacon, “are adorned and celebrated, but are not made to advance; nay, they are frequently vigorous in the hands of their author, and thenceforward degenerate.” This remark of Bacon’s is literally true of Logic; which word will probably suggest to the reader that class of studies at the present time more nearly related than any other to the subject-matter of this book. And in attempting to explain something new and unknown to the reader, we are frequently obliged to make ourselves understood by reference to something already known. In our introduction of the reader to the explanations and substance of the science which we have endeavored to exhibit in the subsequent pages, therefore, we would ask and expect that he has some knowledge of the standard works upon logic, among which that of Archbishop Whately is as able a specimen of the received systems as any; and as it is condense and contains but little irrelevant matter, it is therefore to be preferred to any other. We do not, however, insist that the following pages can not be understood without such previous acquaintance with the works of others. We have endeavored to make this treatise as elementary as possible; so that it may be in itself sufficient to convince and instruct the reader in its various doctrines. Yet such previous acquaintance with the popular works on logic will have brought any one to some appreciation of some of the difficulties in the way, and therefore he will be better prepared for the investigation. And we do positively insist that the reader shall come to the consideration of this book with a trained mind.

On the other hand we do not object to the popular systems of logic because that science has not in the succeeding ages from Aristotle, its author, unraveled itself into greater number of details, or varied its elementary principles. True principles are immutable, and to depart from them is to fall into error. But the popular systems of logic, in our estimation, possess no value except in disputations; they are in fact what Archbishop Whately says of logic in general, “E tirely conversant about language.” That the popular systems of logic make the analysis, and explain the true processes of the mind in reasoning, we do not believe, and therefore we do not regard them as of any value in assisting any person in his search after truth. In driving men by their own words to admit what they already know to be true, i. e., in argumentation, they may be of value; but in making inferences from truths

known to truths unknown, the mind does not proceed upon the principles set forth in the Aristotelian methods, and therefore these methods are of no value to science. In the language of Bacon "They force assent not things." Although the dialects of Aristotle have been before the world for many centuries, yet no one, however well acquainted with the system, has advanced to one new truth in science by the method therein laid down. And if logic is not a peculiar method of reasoning, but THE METHOD upon which all true reasoning proceeds, as contended by Whaley and others, then the great advance which has been made in the sciences must have been made without reasoning at all, or else the method of reasoning is not such as it has been stated and explained by those authors.

But although the popular systems of logic are of no value in assisting to lay the foundations, or to rear the superstructure of the physical, abstract or mental sciences, yet for the purposes of adorning and giving force to speech they are not without value. Archbishop Whately regards rhetoric as the offshoot from logic; in our estimation all that is valuable in the popular systems of logic belongs more properly to rhetoric than to any other science. It is true that whenever we reason i.e. we make inferences from truths known to truths unknown, certain processes take place in the mind, and that these processes are alike in the minds of all men who reason correctly. It is also true that the popular systems of logic explain with tolerable correctness the manner of wording our premises and conclusions in what is called ratiocination. And to go even thus far is an acquisition of no small value. But it is somewhat strange that writers on logic, of the most brilliant talents, who so frequently warn us against the liability of being imposed upon by words, should yet never have penetrated beneath the words to the things that have brought about these words with their manner of usage. Words are used in every science; but no science constructed upon words can touch the limits of things in mental or physical nature. The processes of the mind in reasoning leave no sensible trace behind them; words do not stand as sensible signs of these processes. The mind by its processes forms words, but the processes themselves are at the bottom, and they lie deeper than the words. "A proposition," says Whately, "is defined logically a sentence indicative i. e. affirming or denying; (this excludes commands and questions) SENTENCE being the genus and INDICATIVE the difference, this definition expresses the whole essence; and it relates entirely to the words of a proposition." Any one can easily see that the above definition is grounded entirely upon grammatical distinctions, and as stated by Whately, "It relates entirely to the words of a proposition."

To make a scientific analysis and explain the processes of the mind in reasoning require a different treatment and mode of investigation from that hitherto pursued by writers upon logic. And we may in truth say that in the

pages of this book, we have pursued a method, in the greater part, unattempted heretofore. And certainly the exigencies of the world demand a better philosophy of reasoning than writers heretofore have given. If we look about us in our own country, or travel abroad and observe the various opinions concerning the most common affairs and effects, and notice the zeal with which men pursue the most absurd theories, we will conclude with the great English philosopher, that "the specious meditations, speculations and theories of mankind, are but a kind of insanity, only there is no one to stand by and observe it." It is true that madmen may agree pretty well on many points, and the grave digger in the play thought that Hamlet's madness would not be noticed in England among a people as mad as himself. And notwithstanding the advancement which has been made in the arts, and many of the sciences during the present century, men yet are driven about in their opinions as though there was no certain truth to be obtained, or else they pursue some absurdity as though error and truth both were in effect the same thing. And from these sources of trouble, which set men to travel the wrong road to happiness and true progress, there is no escape for mankind except in the further development of the sciences. But in the words of Bacon, "The present systems of logic are useless for the discovery of the sciences," and "they rather assist in confirming and rendering inveterate the errors founded on vulgar notions than in searching after truth."

"The unassisted hand," however, "and the understanding left to itself possesses but little power. Effects are produced by the means of instruments and helps, which the understanding requires no less than the hand." And in looking after helps for the understanding, we would naturally inquire by what means any one, who had made discoveries in science, had been assisted in his efforts. If any one should see a mathematician calculate the distance to a certain object, or tell the height of a tree without measuring it, and he find the result to be as the mathematician had stated, he would very naturally inquire how such knowledge could be obtained; by what means could such conclusions be reached. And every one knows that the science of mathematics is a most powerful instrument for solving those problems of nature which come within its province. But the science of mathematics itself has been discovered and its truths have been brought to light by certain processes of the mind. And those processes of the mind, which have brought to light some truths in any given case, will, if exercised again in like manner, bring to light other truths of a like nature. The mind certainly possesses the power to gain knowledge by some method, and were this method certainly known and clearly explained, it could be used to advance our knowledge in science, unless all the subjects to which it is applicable are exhausted. But the greater number of the sciences are confessedly yet in their infancy, and the progress, which is made in them, seems to proceed in most instances

rather by chance than by any direction which philosophers have given. The school-boy at the present day studies his logic; but the man who goes forth in the search of truth, throws it away expecting no help from it. Those men, who have advanced science the most, have paid but little regard to those philosophers who have treated of the science of reasoning; while those who have looked and relied upon help and direction from such philosophers, have produced nothing of importance. No person, who has made discoveries in science, will, upon reviewing his experience, acknowledge either that his mind had been led to use habitually the mode of reasoning always to be adopted, or that this mode was suggested to him in given cases by his previous studies of the theoretical systems of reasoning used in the schools. Most persons, indeed, who have advanced science, have been so intent upon their conclusions that, they have not considered the processes of their minds used in gaining those conclusions to be worthy of consideration.

But if the true processes of reasoning were understood, reasoners would certainly be guided to advantage by such knowledge, and they would use this knowledge as an instrument to assist their understanding in solving problems in particular sciences. To assert that there is a science of reasoning and yet to say that this science is of no utility in advancing those sciences, which are built up by reasoning, is absurd. All men admit that the greater part of our knowledge is gained by reasoning, and reasoning certainly does not proceed by chance—but upon some determinate process; and unless these be legitimately pursued our inferences will be fallacies.

Now the science of reasoning ought to inform us when we are in pursuit of any truth; which can be gained by reasoning, what method we must pursue in order to gain that truth: and if the syllogism as explained by the writers upon logic be the method of all true reasoning, then we must find a major and minor premises which will lead us to the truth in question. But according to all the authors upon logic, when we lay down our major premise we virtually assert the conclusion; and hence we must virtually gain the knowledge of the desired truth before we can lay down the premises which shall conduct us to it. We repeat, however, that the popular systems of logic, are not only, not scientific works in themselves, but that they are of no use to science. And hence if we expect to lay sure foundations upon which every science can be built in all their beauties of symmetry, we must look after a better understanding of the reasoning processes than writers upon logic have been able to exhibit hitherto. This we will attempt to do in this book. And we are aware that the task is not only in itself a very difficult one, but that the prejudices of scholars are against us. Bacon's attempt to introduce the inductive system of philosophy has cleared away in some measure the prejudices of many in favor of the Aristotelean methods. But Bacon did not perfect the inductive system, and although he has left here and there very

valuable hints on the other processes of the mind, yet he did not systematize them, and of the true principles of ratiocination he appears to have had no better conceptions than Aristotle and his followers. For the most part, therefore, readers who have formed any opinion upon such matters, will, besides the difficulties of the subject, have to overcome prejudices in their study of this book. A careful study we believe, however, will conquer those prejudices.

Before proceeding to the details of a treatise it is usual with writers to give some definition of the science which they claim to teach in their work, and we will probably be expected to do the same. Some writers have defined Logic to be the art of thinking; others call it the science and also the art of reasoning; and still others consider it to be the science of the laws of thought as thought. For ourselves we do not expect that any definition of Algebra, which can be framed, will assist the student of that science very much in his studies, and therefore, a definition of that or of this science at the outset we do not consider of importance. But besides this we do not wish by a definition to put a band around the inquirers thoughts in the beginning. If a definition convey wrong impressions, it must fetter the mind in its contemplations; and to lead a reader who has not yet studied the science, by a definition to understand the whole drift of the matter would require a full exposition of the definition, i. e. a full treatise upon the definition. We may say, however, that the present treatise is a scientific work, and that the science, whose principles are herein set forth, differs from all other sciences in the respect that it shows the only keys which can be used in unlocking the mysteries of any science. And hence, in general language, this work may be called the philosophy of science. In the title page we have denominated it the Organon of Science—not either from honor or derision of Aristotle's Organon; but because in it we propose to show the instrument or instruments by which sciences are constructed. Bacon called his work the "Novum Organum," and since his time several works bearing that name have appeared, all of which, so far as we know, follow Aristotle rather than Bacon.

The word LOGIC has so many vague meanings in the minds of men at the present day that we have used that word but little in this treatise; although our aim and the aim of most writers upon logic are so far the same that they both propose to lay down some method by which we may be guided and kept from errors. We, however, go much further and assert that our method exhibits the mental foundations of all the sciences and the modes of their construction; and that by the judicious application of our method, whether the thinkers were or shall be conscious of it or not, discoveries in any science always have been made, and always must be made, if made at all. Nor do we believe that we are endeavoring to excite vain hopes when we say that, the thorough understanding of this treatise by the scientific men of the world

can not fail to open to the world a more prosperous era for science than it has had hitherto. And therefore we have the boldness to call upon scientific men and upon all men, who wish for the prosperity and advancement of the human race, to give their serious attention to it, so that intelligence may work out order and happiness in our civilization.

BOOK I.

CHAPTER I.

HIGHEST GENERALIZATION AND FIRST DIVISION.

In every endeavor to prosecute science, we start by dividing off and classifying those entities, which are familiar to us, and which are to be the subjects of our consideration. One class of philosophers, for the purposes which they have in view, divide the objects of earth into the animal, vegetable and mineral kingdoms; and under each of these classes they make numerous subclassifications. The natural philosopher, technically so called, whose object is to ascertain the effects of material masses upon each other, the laws which govern them, and the changes which they undergo without affecting their internal constitutions, commences by classifying matter into solids, fluids and gasses. The astronomer, the chemist, the philologist and historian, have each of them their subjects, objects and classifications. And the necessity of a proper classification, in order to reduce any subject to a science, will readily be perceived from the following consideration: Suppose a certain field to contain several specimens of each of the classes of animals and a certain person to enter it for the purpose of acquiring knowledge, if his mind should not generalize and classify, though he might multiply observations for half a life time he must leave the field eventually without having gained any scientific knowledge. In order to succeed, therefore, the naturalist commences to classify; and his field of observation being animate nature, he seeks for the highest generalization, which his mind can make, and which may embrace in one class, all the objects of his regard. Each subject before him, he perceives, has something in common with every other one, to-wit, animation: and to this highest generalization, he gives the name of animal to distinguish his whole field of research from other things. He then seeks

for other less extensive generalizations, and soon perceives vertebrata, articulata, radiata and molusca. Thus the naturalist proceeds, and by classification alone, he is able to gain a scientific knowledge of the relations existing among animals. In like manner a proper classification of those things about which the laws of mind are concerned in reasoning, is indispensable to the clear understanding of the process employed in acquiring knowledge by reasoning: without a classification as a basis, all before us will be chaos.

But how shall the metaphysician and logician classify? The object, at which he must aim, is to obtain the knowledge of the relations, or rather the knowledge of the results of relations actually existing between the mind itself and all other things, which can be made by the mind the subjects of its cognitions. Now every subject of the mind's cognitions must bear some relation to the mind itself or no result whatever could be produced. And in order to contradistinguish the objects between which the relations exist, from which intellectual results are evolved, the mind itself may be called the EGO and all other things the NON-EGO. The word NON-EGO, however, in this case is not a negative term in meaning, but a positive name for any and everything excepting the ego, or mind itself. The German metaphysicians distinguish the mind itself by "Das Ich," and the French by "Le moi"; and Sir Wm. Hamilton has brought the ego and non-ego into vogue in the English nomenclature. Most persons will know that ego is the Latin personal pronoun corresponding to our personal pronoun I of the first person; ego is more convenient to be used as a noun than our pronoun I, a single letter of the alphabet, and therefore it is used. And we consider these contradistinguishing terms to be apt and useful; for, between the ego and the non-ego, we are to look for the relations and results in question. But yet, how shall we classify the objects of our cognitions in a manner which will evolve and clearly set before us these relations and their results. We cannot clearly set before us these relations by a classification of the various objects comprehended in the non-ego, according to some peculiarities existing inter se, for this does not in a sufficiently apparent manner, involve the ego: and unless both the ego and non-ego be involved there can be no relations existing between them, and no results can be produced. The classification necessary, as a basis of reasoning, must, doubtless, start with the highest generalization; for to plunge 'in medias res,' and classify certain objects, as plantigrade, and others as degitigrade, only points out the comparative anatomy and relations of these objects inter se; and to classify the faculties of the mind into memory, will, imagination, etc., only brings out the relations existing between these faculties. The mind itself, or ego, is not involved in the classification; and consequently the results, springing from the relations of all other things to the mind itself, with their connections on the one hand with the ego, and on the other with the non-ego, can not be appreciated without finding a generalization, which shall comprehend them all.

We must, therefore, seek the highest generalization of both the ego and non-ego that can be made, and taking this for our starting point, descend, divide and classify, in a manner very similar to that pursued by the naturalist.

Now the highest generalization that our mind can make of both the ego and non-ego is EXISTENCE. Existence is a term that may with propriety be applied to any and everything of which we can have any knowledge; each and every shade of thought and feeling, the active principle itself or ego; matter, space, time and the Deity, may each of them, be called an existence; that which can be, and is, is an existence: and this is the highest generalization which we can make of things; it includes the ego and all of the non-ego, the mind itself and everything else, of which we can have cognitions. Now the results about which we are concerned for logical purposes, are evolved from the relations between one existence, our mind itself, and all other existences. The first division, therefore, of existences, in order to keep the relations of the mind to other things in view, must be into the ego and non-ego; these are the two classes of things from whose relations our intellectual results are produced. The highest generalization itself of all things about which we can have any knowledge, can not, indeed, be properly considered a class of THINGS; for, the term EXISTENCE does not distinguish things inter se, but it merely distinguishes, as it were, things from no things, and sets up a state of BEING. But the classification of THINGS into the ego and non-ego certainly puts before our mind, and exhibits to us distinctly the mind of the thinker himself, and all other things which can be the subjects of the thinker's cognitions. And in order to make this classification more clear, we may consider it a little further. We, all of us, believe there are such existences as trees, rocks, water and air, in short, that there is such a thing as matter; we have gained a knowledge of such things in some manner; and we believe that these things are not our mind, but that they exist outside of it, and are what we denominate the non-ego. We believe, also, that there are such things as notions, thoughts, conceptions, feelings, motions, etc., and although these are intimately connected with the ego, and could not exist without it, yet they are not the mind itself, but they are of the non-ego. There is a wide difference between the thoughts, feeling etc., produced, and the active principle, let it be what it may, which is engaged, in some manner in their production. Many of the thoughts of Shakespeare can be found in a book: the active principle, his mind itself, can not be found on paper; his works are the productions of his mind, not his mind itself. But again, we believe our own minds to exist, and that other men have minds. Now my mind is to me the ego, but all other minds in reference to my mind belong to the non-ego; for every person must make his own mind alone the point from which and to which he must make all his bearings in gaining knowledge. But again we believe that there is space, time, eternity and that there is a God; and all

these things are non-ego; my mind itself only for me and your mind only for you are the ego; all other things belong to the non-ego.

Now for further classifications, we have to deal only with the non-ego: for the ego being a single existence is incapable of division and subclassification; but the non-ego is capable of division ad infinitum, and therefore, we may make numerous subclassifications of it. The non-ego, however, must always be subclassified with reference to the ego and not merely with reference to the constituents of the non-ego inter se. The ego and non-ego merge in existence and this must be borne in mind; for, whatever relations, if any, may exist between the earth and the moon, they never could be anything to us unless each of these objects sustain some relation or relations to the ego, my mind for me and your mind for you. That which bears no relation to the ego can not be the subject of our cognitions and it must be to us as though it had no existence; it is only by means of the relations of objects to our minds that we can gain any knowledge of the relations existing between the objects themselves. In our classifications, therefore, it is important to keep in view and take the ego, my mind for me and your mind for you, as the point from which to run to every object of the non-ego.

CHAPTER II.

FACTS AND TRUTHS.

Having in the previous chapter divided existences into two classes in such manner that the relations between them will always involve the mind as one of the things related, we come now to the classification of the non-ego with reference to the ego. And a very obvious division of the non-ego with reference to the ego would be into existences of the past, of the present and of the future. Most of us, no doubt, have had friends whose physical forms have passed away; their forms were existences in the past, but in the present they do not exist; and to-morrow is but a present thought concerning the future. But we must observe that, these divisions only bring out the relations between points of time, in one of which points the ego is now situated; nevertheless, as the ego and non-ego are existences bearing towards each other the relations of time, these divisions, according to the points of time occupied by each, do bring to view the relations between the ego occupying the present point, and those existences of the non-ego occupying the same and different points. But all the existences comprehended in the non-ego may be thrown into another classification, which shall involve the relations existing between the ego and non-ego in other respects than that of time and of that as well.

The first sub-classification, therefore, of the existences of the non-ego, which we will make, will be into FACTS and TRUTHS. And in order that we may understand this division, it is necessary to consider the relations of the

ego merely as an existence among other existences. That which has had a beginning, must have been brought into existence by some anterior existence or existences. We will not stop to argue this point now, for we do not think it will be doubted. And if our minds have not always existed, their very beginnings of existence must be dependencies; and dependent existences come and remain as existences by the influence of that upon which they depend. And when other existences like itself with respect to dependence, surround the ego, the ego and these other existences must be so related to each other that they may act and react upon each other, if each be affected by the other: and each is either affected by the other directly or indirectly, or the one only is effected by the other, or neither the one nor the other is affected by the circumstance of their both being existences. Now between material objects, it is declared to be a universal law of nature, that action and re-action are always equal and in opposite directions. Whether this law be extended to the relations between mind and mind, and between mind and matter, it is not necessary now for us to inquire. But of one thing we must feel assured, that the external non-ego, when its existence is the immediate subject of our cognitions, acts directly or indirectly on the ego. For a tree either acts upon and affects the mind, or to change the expression, the mind is affected by it in some manner, or the mind can have no cognitions of the existence of a tree, and it would be to the mind as though it were not. The mind had a beginning and therefore it is a dependent existence; and an existence, whose coming to be an existence is dependent, must ab initio be passive: and its activity and passivity both, must have been either given to it simultaneously, or the former must have been developed from the later. For, the acting power of a dependent existence can not exist of itself independent of other things, but another or other existences are presupposed to generate it. And if the ego be dependent, its dependence must be upon the external non-ego, otherwise it would be independent; and dependence implies the reception of action. The dependent mind, therefore, is dependent for its existence upon the action of that part of the non-ego, from which its existence came, and for its knowledge upon the action of that part of the external non-ego, of whose existence it gains knowledge.

Now at the first with respect to knowledge, other existences act upon the mind without its inherent energy being exerted. That we are born without any knowledge, will not be doubted by any well informed student since the days of Locke. The mind must exist for a certain period in its inception without consciousness: for to be conscious at all, it must be conscious of something: to be conscious of nothing is to be without consciousness: if consciousness can be contained in mere passivity then a rock can be conscious. But activity is necessary to consciousness: and mental activity must be developed from the mind's passivity by the action of that part of the non-ego,

upon which the mind's dependence in this respect consists. For the power to receive an action must be contemporaneous with the mind's existence: but the mind must exist in the world before it can be acted upon by any power, other than that which created its being before it was really a mind. When, therefore, the ego first comes into the relations of that part of the non-ego, from which its existence was not derived, it must first be acted upon and act in response before it can be conscious of that part of the non-ego. And when we reflect that the external non-ego affects the mind only through the senses, and that in the foetal state, all these senses, even that of touch in a great measure at least, are secured against external impressions, we can not doubt that the mind at first is unconscious of an external world. And the only other things of which it could be conscious, are the action or actions of the power which caused it to exist, and of its own existence. Now the action of that existence or of those existences which created the mind, must still continue to be exerted, or the ego becomes either an independent existence or a non-entity. But we have shown the mind to be dependent, if it had a beginning; and therefore we may with matured faculties appeal to our consciousness respecting the action of that creating power, and all persons will say that they are entirely unconscious of the action of that power which prolongs our existence. It is, however, commonly said that we are conscious of our own existence, i. e., that the ego is conscious of itself per se; but we regard this as an error. For unless the mind act, it can not be conscious at all: and when it does act, it is conscious of its acts, states and feelings; but of itself per se it is not conscious. Each person can test the truth of this by his own consciousness. And if the mind at first be unconscious of the action of the external world through the senses, and also unconsciousness of the powers which prolong our existence and unconsciousness too of its own existence per se, it must at first be without consciousness. The mind, indeed, can be conscious of its own acts and feelings; but independently of the action of other existences upon it, it can not begin to act or to feel.

Now we find that a material body made up of bones, muscles, cartilage, membranes, nerves etc., all of which belong to the non-ego, contains the mind. This body is related both to other existences without and to the mind within: it is a medium between the mind and existences external to itself. And the first effect produced upon the ego by or through this body gives the mind merely that state of activity which we call intensified passivity. The mind does not yet NOTICE; but it possesses more than mere passivity: it does not yet put forth its energy in any definite direction, but it possesses energy. But in a little time after birth, by being continually acted upon by the external world through the senses, the mind's intensification is increased, and its energies start in definite directions, and then it notices. But it merely notices. By the eye, the ear and the other senses, it notices existences: but the WHERE,

the WHEN, the WHAT, or the WHY, it does not KNOW. But in a little more time, the mind begins to discriminate and then it begins to know and to have knowledge.

Without the power to discriminate, we could know nothing, although we might notice some things: and the possibility of discriminating lies in the relations between the non-ego and the ego. Now the only relations, which can exist with reference to the ego, between the existences among which the ego is placed, and with which the ego itself must be contemplated, are those between the ego and external non-ego directly, those between one external object and another of the non-ego indirectly through the ego, those between one external and one internal object of the non-ego through the ego, and those between one internal object and another of the non-ego. From each of these relations and from them only can we discriminate and gain knowledge. From the relations existing between the ego and the external non-ego directly, we have the action of the non-ego upon the ego, and the response of the mind itself in a directly opposite direction to the one received. This is the mere noticing of an object by the mind and it constitutes a FACT. But if in the noticing of an external object of the non-ego, which is a FACT, the mind also notices its own act, which, we think, is the case, here is another thing noticed, a FACT different from the former, and these two FACTS may be compared. And let the same process be repeated with the same external object of the non-ego, and we have a relation between two acts of the mind itself, between two internal objects of the non-ego; and also a relation between each act of the mind and the external object. And hence among these relations, three comparisons may be made, viz., between each act of the mind and the external object, and between the two mental acts inter se: and from either of these comparisons, the mind can gain knowledge. From the comparison between the action of an external object of the non-ego upon the ego and the act of the mind itself in return, we gain the knowledge, that the act of the mind itself and the action of the external object are separate existences: and from the comparison between two acts of the mind itself, we can also discriminate and gain the knowledge of separate existences. For two acts of the mind in the same direction can not be simultaneous: and the interval of time, however small, forms a relation by which the mind can discriminate and separate internal existences. Separate existences hereafter we will call HETERA. (Greek—heteros, a, on—others). We use the neuter plural of the Greek adjective as a noun, meaning other things—separate existences. And hence the evolution of hetera by the mind is the inception of human knowledge. By the mere noticing of an object, the mind indeed acts, but can know nothing, because one object per se can not be compared and discriminated. But if the mind notices its own acts in noticing external influences and compares them with that of the thing noticed, from the rela-

tion existing between the two, the mind can evolve the knowledge of heteria. And we must here remark again, that the mind does not and can not notice itself. Its acts, states and feelings, it can notice: but the knowledge of its own existence, as a potential mind per se, is gained only by comparison.

Now things merely noticed by the mind we call FACTS: the knowledge gained by the comparison of noticed existences, we call TRUTH: and this is our first classification of the existences of the non-ego. FACTS then, are existences, each one of which is noticed by a single act of the mind and without comparison: truths are the results of comparisons made by the mind between facts and also between truths themselves. Now FACTS are all comprehended in the non-ego, and of them we may make two classes: the one class having their WHERE without and the other having their WHERE within the ego. The first of these classes we will call PERCEPTIONAL and the second SELFCONSCIOUS FACTS. And although neither of these terms are in common use in our language, we think we have the right to adapt terms to our own purposes. From the Latin fractio, we have fraction, from which the adjective fractional is constructed: and from perceptio, we have perception, from which in like manner perceptual may be made in harmony with the principles of our language. And thus, also, we may deal with conscio and prefix self.

And each of these classes of facts may again be divided into five sub-classes. Perceptual facts are naturally subclassified into the five classes, viz.: visual and auricular facts, facts of touch, of taste and of scent. And hence one external aggregate existence—and by aggregate existence we mean an existence to which we can apply our organs of touch, of taste, of smell, of sight and hearing—may contain five perceptual facts or external noticeable existences. Such an existence as RED, or an existence to which we can apply but one specific organ of sense, we call a simple existence and not an aggregate one. But two aggregate existences, then, will contain ten perceptual facts. And if each fact of the same aggregate existence, be compared with the others, there will be ten comparisons of facts inter se of the same aggregate existence. And if we compare each fact in an aggregate existence with each fact in another aggregate existence, we will have twenty-five comparisons. And hence two aggregate existences contain ten facts and afford forty-five comparisons, from all of which truths can be gained.

CHAPTER III.

CONSCIOUS TRUTHS.

In the preceding chapter we explained what we mean by FACTS and endeavored to show to what existences we apply that term. We showed that those existences which we call FACTS; in and by themselves separately considered, make no part of our knowledge; but that they are the foundations

and pre-existent substrata upon which all our knowledge stands and from which it springs. All knowledge lies in relations, and the mind evolves it by comparisons. Were a person so brought into life that he could see the sun, i. e., notice this perceptive fact, but notice nothing else, i. e., have no self-conscious fact, he could not know that the sun exists. We can not say that the sun exists without having the knowledge of existence. For, the phrase "The sun exists," or "The sun is," is equivalent to this, viz.: the sun is an existence. And unless we first have the knowledge of existence, we can not know the sun to be one: not a single FACT but FACTS must come to the mind before knowledge begins. And when the mind first notices a perceptive fact, there is also always lodged in it a self-conscious one; these facts, the one perceptive and the other self-conscious always enter the mind in a binary manner. For, as we have already said, the ego unconscious of itself per se, takes its place among other existences to be acted upon and to act in return. And these perceptive and self-conscious facts keep coming in a binary manner repeatedly before the mind compares them at all: but when it does once make the comparison, the knowledge of separate existence is evolved. This knowledge we call conscious truth. And hence we say that we are conscious of an existence though the knowledge of an existence be not a fact to us, but a truth evolved from the relation of facts: the fact of an existence per se is noticed but not known by us.

The relation of perceptive and self-conscious facts is necessary to the beginning of consciousness. For, as already said, to be conscious implies to be conscious of something, and to be conscious of nothing is to be without consciousness; and the human mind had a beginning of existence and it is a dependent being. And although, indeed, we can not tell by the proofs which nature offers, but that the *materia mentis*, so to speak, may have always existed, and that at the first it may have been inclosed within a human body, and afterwards handed down from generation to generation; yet that there was a time when our consciousness did not exist, is clear. For, the *materia mentis*, let it be what it may, could not, per se, by its own inherent power separated and independent of all things else in the universe, be conscious of anything except itself per se. And although the mind be conscious of its acts, states and feelings, yet that it is not conscious of itself, i. e., not conscious of the FACT of a *materia mentis*, our own consciousness teaches us. And if the mind be not conscious of the fact of its existence, or to use a phraseology more tangible to some minds, if the mind can not feel itself per se, it must be a dependent being, and its dependence must be a dependence in every respect at least except existence alone. And that the *materia mentis*, in such relations as entitle it to be called a human mind had a beginning can not be denied: and hence its consciousness in those relations must have had a beginning also. And as the human mind is inclosed within a body, were

this body impervious to the action of all external things, the mind must continue unconscious. And although it is often said that consciousness is the very thing that distinguishes animate life: yet the lack of actual consciousness does not establish the lack of potential consciousness or the nonentity of mind. Consciousness is not the mind itself: the *materia mentis* must first exist before consciousness can. And if, as we have shown, the mind in order to be conscious, must be conscious of something, that something of which it is conscious, must be brought to the mind itself by the external non-ego: otherwise, the human mind could rear a structure of knowledge from out of itself and independently of all things else in the universe. Consciousness, therefore, as it can not exist without a mind to contain it, so likewise it can not exist in the human mind independent of all things except the mind: without the non-ego the ego could not be conscious.

Now there is in man a *materia mentis*, or an immaterial substance, or if you please and as some suppose an arrangement of physical organs in some manner so that the arrangement affords the conditions necessary to become conscious when acted upon: we start no question respecting either of those or of any theories. What may be the essence of mind, we do not know, but whatever it may be, we find it, in a proper organization, to be capable of knowledge; and our inquiry here is with reference to this knowledge. And the first knowledge, which the mind gains, is CONSCIOUS TRUTH. And if consciousness depend upon the relations of facts, i. e., upon existences which are inter se hetera, it must spring from those relations. We may say, that the mind has knowledge of something. This sentence contains the mention of three existences viz.: mind, knowledge and thing. We may say that the mind is conscious of something; and this sentence contains mind, consciousness and thing. And if, as we have shown, the mind notices its acts, but not itself, and consciousness be dependent for its existence, then, if the later sentence be true, consciousness must have been evolved from the relation of the action of the MIND, and that of the THING. An object of the non-ego affects the *materia mentis*, the mind acts; and from the relation of the effect produced upon the *materia mentis*, and the returned action of the mind, spring consciousness or the knowledge of existence. Consciousness is the result of relations and it is evolved from FACTS. When we say that we know that stove is not an act of our minds, because we are conscious of this, we state what is not true. We become conscious of the existence of an act of mind and of a stove, and the judgment then discriminates between the two by comparison. Consciousness is merely the knowledge of existence; and the thing or existence of which we are conscious, we call a conscious truth.

Now we have shown that there are perceptive and self-conscious facts; there will be evolved therefore, from the relations of these two classes, conscious truths grounded in the non-ego and also conscious truths grounded

in the ego. And as numerous as the perceptive and self-conscious facts may be, so numerous will be the conscious truths. For every relation between perceptive and self-conscious facts evolves two CONSCIOUS TRUTHS. The relation between the perceptive fact of a tree and the self-conscious fact of the mind's act in noticing that tree evolves two conscious truths, the one being external and the other internal. From the relations of self-conscious facts inter se, however, or from the relation of perceptive facts inter se, conscious truths can not spring. From the relations of perceptive and self-conscious facts, spring conscious truths, and then these conscious truths can be compared promiscuously. CONSCIOUS truths, therefore, like perceptive and self-conscious facts, upon which they immediately depend, come to the mind in a binary manner.

Now by each of the five senses, the mind notices perceptive facts: when these facts by their relation to self-conscious ones, rise into consciousness, they become conscious truths which are grounded in the non-ego. So likewise when self-conscious facts from their relation to perceptive ones, rise into consciousness, they become conscious truths, which are grounded in the ego. There are, then, two great classes of conscious truths, viz: conscious truths grounded in the non-ego, and conscious truths grounded in the ego. But that the one class is grounded in the ego and the other in the non-ego, is not determined by consciousness, i. e., we are not conscious of that, but this knowledge arises from an act of judgment in comparing two conscious truths, i. e., two existences of which we have become conscious.

Now it is said by some philosophers, that the mind does not occupy space, i. e., that space is not necessary, not one of the conditions of its existence. But nothing certainly can be more absurd: for that, which does not exist ANYWHERE, can have no existence. Because we can not tell the precise WHERE in which it does exist, does not prove that it has not a WHERE in which to exist. That, which has an existence NOWHERE, has no existence at all: and every WHERE is a WHERE in space. The ego exists SOMEWHERE and in this WHERE lie the conscious truths grounded in the ego: the non-ego exists somewhere and in this where lie the conscious truths grounded in the non-ego: the wheres of the ego and of the external non-ego are hetera of space. Now we must recollect that the conscious truths grounded in the ego and those grounded in the non-ego come into existence simultaneously; the only things therefore, which the mind can discriminate, between conscious truths grounded in the ego and conscious truths grounded in the non-ego, merely as existences, are the WHEREs occupied by each, i. e., the wheres can be discriminated into hetera. We classify, therefore, all conscious truths into conscious truths grounded in the ego, and conscious truths grounded in the non-ego: and that these two classes of truths respectively are thus grounded, the mind determines by HETERATING their WHEREs. Each of these

great classes of conscious truths may be again subclassified: The conscious truths grounded in the external non-ego are classified into conscious truths of touch, of taste, of color, of scent and of sound; and the conscious truths grounded in the ego, into hearing, seeing, feeling, smelling and tasting. All these, both those grounded in the non-ego, and those grounded in the ego, are inter se hetera. A sound is not the same thing as hearing, nor a scent the same as a sound; any two of the same class or of different classes, are hetera. And hence of the conscious truths grounded in the non-ego there are five classes, and of the conscious truths grounded in the ego, there are five classes, making in all ten HETERICAL subclasses of conscious truths.

CHAPTER IV.

NOMINAL AND PROPOSITIONAL TRUTHS.

In the last chapter we endeavored to show what we mean by conscious truths. We do not mean my conscious truths, truths which possess consciousness, but existences of whose entity we become conscious. And we showed that we gain the knowledge of conscious truths by being able to separate the external and internal existences of the non-ego into hetera. This is the first step in the acquisition of knowledge. And were we not able to do this, all would be chaos; but this once done, chaos breaks and order takes a beginning: and then we proceed further and discriminate internal existences inter se, and also external existences inter se into hetera. But, as yet, we know heterical existences, we have the knowledge of existence merely as existence; and merely as existence, existences are all alike. A sound, a taste, a color, etc., merely as existences are hetera but alike; they are, as existences, heterical similia (Neuter plural of Latin; similis, e—things resembling each other).

But sound, taste, scent, color and touch, being existences grounded in the external non-ego, may be further discriminated by the different modes or manners by which they are related to the ego. And hearing, seeing, smelling, tasting and feeling being existences grounded in the ego may also be discriminated inter se by the modes or manner by which they are related to the external non-ego. The manner of receiving visual impressions and seeing is different from that of receiving aricular impressions and hearing. And this difference of mode or manner, whether there be any other difference or not, distinguishes the five classes of conscious existences grounded the non-ego inter se, and also the five classes of conscious existences grounded in the ego inter se. These modes or manners by which the mind is brought into relations with the external non-ego, belong to our physical organizations, and inter se they are deferentia (Neuter plural of Latin differens, ens—things differing).

By DIFFERENTIA we do not mean difference, but things differing, hetera

unlike. The difference between two feet and one foot is one foot: the difference in area between a parallelogram and triangle of the same base and altitude is one-half the area of the parallelogram: but the difference between red and green can not be pointed out. The difference lies in the causes of these effects upon the mind; but what those causes are, we do not understand sufficiently, so that we can contemplate them otherwise than by the effects themselves, which we can only discriminate into things differing—differentia. If we resolve a ray of light into its elements by the prismatic spectrum, and then from different combinations of elements, each combination having one element at least in it the same as in the others; we find different colors to result, the difference between these combinations, is the additional element or elements in the one more than in another: but the difference between the effects per se of these combinations upon the mind, we can not point out. That these effects per se are differentia, hetera unlike, we know; but that is all we know about them per se.

Now had it been possible for man to have become conscious of only one existence, he never would have invented a name for that existence. For everything which has a name, has received that name to distinguish the result of a heteration of a differentiation or of a comparison of things. Suppose, for instance, that every object of vision had possessed but one color: no distinguishing name then for any color to distinguish it from others, could have been introduced into language. For the word "color," would have expressed all the knowledge that man could have had in that regard. And although this existence (color) would have arisen into consciousness: yet the only necessity in a name for it, would have been to distinguish it from conscious truths of the other senses. And unless men became conscious of the very essence of existence they could by making some possible discrimination give names only to distinguish existences inter se. And supposing now, all the senses excepting sight to be wanting, and all objects to vision to possess but one color, then there would be no other existences grounded in the non-ego to discriminate inter se, and the words SEEING and COLOR would have been sufficient to discriminate the parts of man's knowledge. But suppose now that along with the one color, one existence of sound should rise into consciousness, here now is an existence of a different mode, possessing a different relation toward the ego from color. There is, indeed, no assignable difference within our knowledge between a color and a sound per se, they are simply differentia, hetera unlike; and their modes of relation to the ego are differentia: but the difference between hearing and seeing per se cannot be pointed out. The differential modes of relation, give us the knowledge of the differentia, sound and color. And now, upon the above supposition, we know one sound and one color, and know these two existences to be differentia: and to distinguish these two existences inter se by words, two names

are necessary. A name for the one existence alone, will not answer to enable us to mention the other. If we should call the one color, not color might stand for the sound. But suppose now a scent also to rise into consciousness: we have now three differentia: and if we wish to speak of them, we must have three distinguishing terms, one for each: and so on through the senses.

And hence we see that there will be five generic names in every language, which has attained to any perfection, to distinguish the five differentia of conscious truths grounded in the non-ego. These names are signs of the results of the mind's discriminations by modes of relation among conscious truths grounded in the non-ego. A like discrimination is also made with like results among conscious truths grounded in the ego. But in giving these names, men are not naming FACTS, nor are they naming conscious truths per se; but they are giving names to distinguish conscious truths inter se. FACTS grounded in the non-ego per se, have no names to distinguish them inter se: conscious truths per se have but one common name, to-wit, existence; but conscious truths, which are inter se differentia, have five names for those grounded in the non-ego, and five names for those grounded in the ego: each of the differentia is in language distinguished from the others by a name. These truths spoken of, which are inter se differentia, and grounded in the ego and in the non-ego, we will call NOMINAL truths: because they are the first truths distinguished by differential names. The NOMINAL truths, then, are sound, taste, color, touch, scent and the hearing, seeing, feeling, smelling and tasting: all these are inter se differentia. We do not mean, however, that these truths were historically the first truths named. The progenitors of our race would be likely to give names to aggregate existences first, as they would come in contact and feel deeply interested in them from the beginning. But philosophically, when attempting to reduce our knowledge to scientific order, NOMINAL truths come up next after conscious truths and they are the first truths distinguished by differential names.

Now proceeding with our inquiry, as we have called differential conscious truths, nominal truths; so the truths gained by differentiating nominal truths inter se, we will call primary propositional truths: because they are the first ones that can be exhibited in propositions in which the words NO, NONE and NOT do not occur, and in which the subject and predicate are not represented by the same name, as RED is a color. And for the present, we will dismiss from our consideration, those truths grounded in the ego, and consider those only, which are grounded in the non-ego. Suppose all the existences of vision presented to our eyes for twenty years of our life, to have had but one color, green for instance: and supposing all of the senses to exist in a healthy state, at the end of that period, we would have the nominal truth of color, and some name to distinguish it from the nominal truths

of the other senses: suppose this name to be COLOR. And suppose that another existence, RED for instance, should then become a conscious truth. Now if we should compare this new existence with all the others of which we had any knowledge, excepting green—the first color, we would perceive that it was not on the same scale of truths, like any of them in any respect. As a conscious truth it is like them all; for all of them are conscious truths. But as a nominal truth, a further consideration and discrimination, this new existence has nothing in common with any of them. But if we compare THIS RED with that GREEN, we perceive that they both agree in their modes of relation to the ego; and it was because the modes of relation to the ego are differentia that the conscious truths of sound, taste, scent, etc., could be discriminated into differentia—into nominal truths. But in the case of red and green, the modes of relation to the ego are not differentia, but similia, and hence red and green, as conscious truths, can not be discriminated at all into differential nominal truths; but we must proceed further and discriminate inter se nominal truths (to which both red and green belong, and therefore the word color is applicable to both), into primary propositional truths. RED is discriminated from the conscious truths of the other senses, in the same manner that green is, and the name color may be applied to both and it sufficiently distinguishes them from the other nominal truths; but it does not distinguish RED and GREEN inter se. And to do this we must necessarily discriminate colors. This we are able to do. And the reason that we are able to discriminate colors, lies not in their modes of relation to the ego, but in causes, which are differentia working through modes, which are similia: the modes of relation to the ego are similia, but the relations themselves are differentia: and to distinguish these relations inter se two names must be used. Red and green, therefore, as nominal truths, are both distinguished in language by the name color; as primary propositional truths, the one is distinguished by the name RED and the other by GREEN. And hence we can say that this color, this nominal truth distinguished by its mode, is among truths of the same mode, distinguished by the name red: this color is RED. And if we add another color to our list, we must deal with it in like manner, and simulate it with the nominal truths of color, and then differentiate these simulated nominal truths into primary propositional truths; and so on through the colors. And if we now call color a genus, as is generally done by logicians, we will then have species of color. And thus we may deal with scents, sounds, tastes and feelings.

And hence we see that primary propositional truths arise by comparing, and generically simulating and specifically differentiating nominal truths. And these primary propositional truths, which as primary propositional truths agree in every respect, will of course, be classed together, i. e., will have a common name for each and every one of the individuals thus alike:

just as all nominal truths inter se similia, will, as nominal truths, have a common name. Take the primary propositional truth RED, and suppose two heterical REDS to be before us: now two heterical reds as primary propositional truths, are exactly alike in every respect, starting from the FACTS, which lie at the foundations of them. They are both perceptive FACTS: both are conscious truths grounded in the non-ego, both are nominal truths, and both are primary propositional truths: but we can carry our discrimination no further. As primary propositional truths, they are alike in every respect in every step from FACTS: and could we not at the second step, existences grounded in the non-ego, discriminate them into HETERA, they would be to us the same thing. And in this manner are sounds, colors, tastes, scents and touches divided and classified.

The nominal truths of sound are divided into musical and non-musical. And the primary propositional truths of musical sound are again divided into rhythmic, melodies and dynamics: these last are secondary propositional truths. Non-musical sounds too are frequently subclassified by calling to our mind and connecting with them some object which is supposed to produce them, or some state or feeling of the mind itself, which certain objects produce; as vocal, nasal, pleasant, dismal, deathly sounds, and so on. But there are, no doubt, thousands of truths perceived by the mind without names to distinguish them. For the colors, which are differentia, and the sounds which are differentia and so on, are very numerous; and only the very appreciable and marked differentia receive distinguishing names. Now conscious truths, nominal truths, primary and secondary propositional truths, exhaust our knowledge of those simple existences, which we will have occasion hereafter to call facial gregaria.

CHAPTER V.

ORDINAL, CARDINAL AND TEMPORAL TRUTHS, AND TIME AND SPACE.

Having in the last chapter treated of those existences, which we will have occasion to use again in our inquiries under the name of facial gregaria, we must now proceed to classify still other truths, which enter into our daily concerns of life, and from which we continually reason. We have already shown hetera to lie at the very foundation of our knowledge. And although the UNIT is the first of the series of cardinal numbers and the base of the system, yet duality or plurality is necessary to our knowledge of the unit. Without the knowledge of two existences at least, we could not have the knowledge of the unit. For, the knowledge of ONE springs from numerical relations; and with one existence per se there can be no numerical relations. Now we have already seen that, differentia receive distinguishing names. But hetera also receive names to distinguish them inter se. If we compare one conscious truth with another, and cannot discriminate them into nominal

truths, i. e., into differentia, the only way that is left for us to distinguish them at all by names, is to mark them first, second, third, etc., and this result is accomplished by distinguishing existences merely into hetera and marking the individuals. These truths, therefore, we call ORDINAL truths. They come to our minds in point of time at an early period of our knowledge: but they may not receive names to set them out clearly for a long time afterwards. Ordinal truths are simply the relations of separate existences as existences and their names distinguish the individuals inter se. And hence these names may be applied to anything, just as we may call anything of which we have knowledge, an existence. And in point of time the ordinal truths or numbers, philosophically considered, must come to our minds before the cardinal truths or numbers. And historically, this appears to have been the case. We find the ancient Jews, Greeks and Romans, using for their notation the first ten letters of the alphabet, which upon reflection will be seen to express much better the ordinal than the cardinal numbers, and for which purpose they were most probably used at the first, and for which they are now with us exclusively used.

And after the ordinal numbers or truths are obtained, we have but to compound or colligate them and name the colligations (for in nature they will be similia hetera unlike) and we will then have the cardinal truths or numbers. Cardinal truths, therefore, are colligations of hetera with a deference inter se of one, and they are distinguished in language by the names one, two, three, etc. And as each colligation is a colligation merely of hetera, the distinguishing name given to any colligation may be given to a like colligation of things differing in nature from the first, as two men, two horses, etc. The abstract nature and applicability of numbers to any and everything, is owing to the circumstance, that they are names of hetera, which do not take into consideration, in any manner, differentia in nature, but which merely represent heterical existences. When, however, we apply these numerical names to objects in the concrete, the objects must be heterical similia. We can say that a potato and a horse are two existences, but we can not place after the word two any differential name by which we can express, in the concrete, the numerical sum of a horse and a potato.

But again: we have already seen that facts, the one perceptive and the other self-conscious, enter the mind in a binary manner, and from their relations, acts of the mind itself become conscious truths, known existences. And conscious truths grounded in the ego may be compared inter se, and from their relations another class of truths may be evolved. If two acts of the mind in the same mode and direction, be discriminated, we will have the temporal truths of once, twice, thrice, etc. Whether a man can hear, see, smell, etc., all at the same time, which is probable, we will not discuss. But that a man can not see or hear, i. e., that the mind cannot act in the same

mode and direction in either hearing or seeing twice at one and the same time is evident. Place an object before you and look at it, and then after having taken your eyes away from it, look at it again, and you will not say that you have looked at it twice at one and the same time. The comparison, therefore, of two conscious truths inter se similia, grounded in the ego evolves the temporal truths of once, twice etc.

But again: if we resolve existences grounded in the non-ego into hetera, we will, of course, perceive a plurality of existences. And if the modes of relation to the ego, of two existences so resolved at the same time, be the same, we must perceive that the two existences do not occupy the same WHERE, for if they did we could not, at the same time, resolve them into hetera. RED; for instance, which occupies but one point, can not at the same time be resolved into hetera, into separate existences, into two REDS. HETERICAL existences grounded in the non-ego, which are related to the ego in like modes, necessarily occupy heterical WHEREs. Each of these WHEREs may be but a single point, which can not be resolved into hetera; but the two wheres must be separate, and if they be separate, that which separates them we call SPACE. Space is a truth which forms a class of truths by itself alone. Wheres are necessarily resolved into hetera, when we resolve existences grounded in the non-ego into hetera, i. e., existences grounded in the non-ego can not be so resolved without heterical WHEREs. When we resolve existences on the other hand, which are grounded in the ego and produced by the ego's action in the same mode and direction, into hetera, we necessarily resolve TIMES into hetera. Time also is a truth, which forms a class of truths by itself. Mr. Hume derives our knowledge of space from color. And if a color cover sufficient space to be resolved into two or more SOMEWHEREs, space will be evolved from the relation of those WHEREs: but if only a single point of color so minute as to be incapable of being so resolved, be presented, no knowledge of space can be gained from such point per se. Mr. Locke obtains all our knowledge of space from both touch and color, and this may also be done in the manner we have stated. Sir Wm. Hamilton calls space "A native idea of the mind," which expression seems to have no meaning.

We have now shown how we derive and classify our knowledge of colors, tastes, scents, touches and sounds, and of acts of the mind itself into hetera, of ordinal, cardinal and temporal numbers, and of time and space. And it will be seen that EXISTENCE, not as a class distinguished from other THINGS, but as the state of being in contradistinction to non-entity, stands at the head of our inquiries. Existences are then divided into perceptual and self-conscious facts, and from the relations of these we evolved conscious truths, our first class of truths. We then found some conscious truths to be grounded in the ego and others in the non-ego, and in each of these classes

we found NOMINAL truths, so called because they are the first truths which receive differential names. From the relations of nominal truths inter se, we then evolved primary propositional truths, so called because they are the first truths which can be used in propositions in which the words NO, NONE and NOT, do not occur, and in which the subject and predicate terms are not the same name. We then evolved secondary propositional truths, and saw that we had exhausted those simple existences which hereafter we will call facial gregaria. We then evolved the ordinal, cardinal and temporal numbers and time and space. And we must still proceed further with our inquiries before we commence where logicians have usually commenced in treating of the reasoning processes. But if the reader will have patience to follow us in our preliminary inquiries, we believe, he will be able when we come to treat of propositions and the syllogism, to understand the whole matter, and to escape from the obscurities and perplexities, which in our opinion, have hitherto surrounded those subjects.

CHAPTER VI.

CLASSIFICATION OF AGGREGATE EXISTENCES AND OTHER TRUTHS.

Having already considered those simple existences grounded in the non-ego, which we shall call facial gregaria, we come now to the contemplation of aggregate existences. We may find a color, a sound, a taste, a touch and a scent, all situated in one location. Two existences grounded in the non-ego and related to the ego by the same mode, can not occupy the same WHERE at one and the same time: for if they do, the existences can not be hetera. Thus: two colors can not exist in the same WHERE, nor two sounds, nor tastes, etc., at the same time. But the five nominal truths grounded in the non-ego, nevertheless, may all be found co-existing at the same time in the same WHERE and forming the facial gregaria of an aggregate existence (Gregarius, a, um; gregaria, neuter plural—things in a herd). And by an aggregate existence we mean an existence composed and made up of simple existences; as the leaf of a rose, iron, snow, a stone, water, etc. These aggregate existences grounded in the non-ego possess facial gregaria, some, if not all of the nominal truths grounded in the non-ego.

But aggregate existences, besides the facial gregaria, have also capacial gregaria, i. e., capacities to receive and give effects among themselves. If we move two heterical and aggregate existences towards each other, we find that both can not be made to occupy the same WHERE in space at the same time; one of them must necessarily exclude from its WHERE, the other, or they both could not remain hetera. This capacial gregarium of aggregate existences is called impenetrability, and is said to be one of the primary properties of matter. And each particle of matter must necessarily have a where in space and without a where it must cease to be an existence. Impenetrability,

therefore, is one of the essential capacial gregaria of aggregate existences: if matter did not possess impenetrability each particle might annihilate its neighbor until the earth became a non-entity. And another essential capacial gregarium of aggregate existences is form or figure.

But after we have gained a knowledge of matter, i. e., of aggregate existences, we readily perceive that in some matter the particles cohere rigidly, while in others they move freely among themselves. This capacial gregarium of the one and that of the other are inter se differentia: and if we distinguish these gregaria inter se we will have the classes, solids and fluids. Then again fluids may be discriminated by their facial and capacial gregaria: one will not have a like color with another, and their tastes may be differentia: a volume of one may be tried in a balance with an equal volume of another, and their specific gravities be found to differ: heat may be applied, and fluids be found to differ in the degrees of heat necessary, *ceteris paribus*, to make them boil, etc. And wherever the mind can discriminate into differentia, it will form classes of fluids; and those which are not to us differentia, may be called by one and the same name. The knowledge of all classes of fluids is gained by differentiating their gregaria either facial or capacial: capacial as well as facial gregaria being truths grounded in the non-ego.

And when men begin to examine matter closely, they find that the particles composing one bulk may be analyzed, i. e., discriminated into differentia. And hence they form classes of what they call elementary substances, i. e., aggregate existences, the particles of which can be discriminated into hetera, but not into differentia. The ancients knew but four elements, viz: earth, air, fire and water: man has since found a great many more elementary differentia. And every differentiation, that the mind can make, throws new light upon the world and adds new truths to our store of knowledge of the elements. Now the number of facial gregaria that matter may possess, so far as we can know, when expressed in the classes of nominal truths, is five. Each of these five classes, however, are divided into numerous primary propositional truths, which have names, and besides these there are various other classes of which we have knowledge but for which we have no names. But the number of capacial gregaria of matter is found out slowly, one after another: and where the number ends we can not even guess. Each generation to come may find out new capacities of matter, and when they do, they will of course make new classifications according to the differentia discovered. We have matter, now, classified by its specific gravity, its attraction of cohesion, its friability, its ductility, its malleability, its compressibility, its effects received and produced among existences, etc. Any capacial gregaria, which are inter se differentia, may produce classes of matter. Chemistry is a succession of differentiations of elements and compounds, i. e., capacial gregaria discovered by experiment. And what is very strange, the mineral

enter into compounds in a binary manner, as truths are compounded, so to speak, in a proposition as we shall see by and by. Thus: carbon and oxygen unite and form carbonic acid: hydrogen and nitrogen unite and form amonia: and then the carbonic acid and amonia unite and form the carbonate of amonia. Now the mental process of simulating and differentiating hetera, gives us all the classes, which we possess, of the different kinds of compounds and elements. The classification of matter by differentiating its capacial gregaria, so far as it has been accomplished, may be found in works on chemistry and materia medica. And we must perceive that aggregate existences when stript of their facial and capacial gregaria, are unknown to us. The gregaria are the only things of which we have any knowledge through the senses. That which lies behind the gregaria are merely inferences drawn from the gregaria.

Now after knowledge has increased and language been invented to express it, the science of grammar takes its rise. Men begin to simulate and differentiate words. The parts of speech are classified by differentiating the intentions of the mind in using different words, i. e., by the functions of words. The principles of the declensions of nouns and adjectives, and of the conjugations and inflections of verbs are obtained in the same manner. The knowledge of tense is gained by the discrimination of times into hetera: of modes by the differentiation of manners and so on.

The same mental process also obtains in Botany. The botanist differentiates, cotyledons, radicles, plumules, etc., and as the plants grow he finds buds, which he in like manner classifies into auxillary, accessory, adventitious, latent and so on, he also differentiates the leaves and give distinguishing names to each class. The whole classification of botany, shows, that the human mind has been dealing with every part of the plant by simulating and differentiating.

And if we look into Zoology, the same mental process meets us at the threshold. Vertebrated, radiata, articulata, rumenants, pachydermeta, plantigrade, etc., are classes obtained by the differentiation of truths. And this can easily be shown to be the case with ethnology, entomology, mineralogy, anatomy and all of the natural sciences. And hence, each of those sciences is also a mental philosophy giving us the classifications of as many truths as the particular natural science contemplates. Accepting therefore the classifications of the several natural sciences and making them our own, we will proceed to consider other truths, which come to our knowledge from other sources.

After having obtained the knowledge of space and matter, we may easily get the truth of extension. Extension, indeed, independent of everything else has no existence: it is not a conscious truth. We speak of the extension of space and that of matter: but had there existed nothing extended

extension could have made no part of our knowledge. And whatever is extended must be so extended that two points in space, two somewheres, can be discriminated by the mind. And hence extension when applied to matter means consecutive and contiguous points, which can be discriminated. And in every other sense, the word is misapplied; and it is thus when we use extension as synonymous with space. The proper meaning of the term extension is the stretching out of something. And if we take two points and consider the space between them, and then remove one of the points further from the other, the space between them will be extended. So if we consider a colored point on paper, the enlargement of that point will extend the area of the color. A mere mathematical point can not give us the knowledge of extension: but two mathematical points separated from each other, can give us the knowledge of the extension of space. Our knowledge of extension is gained by the discrimination of heterical points located in something in space, or in space itself. The consecutive points must all be in some existence of the non-ego: for extension is a truth gained by the comparison of truths grounded in the non-ego. Extension, like time and space, forms of itself but one truth and a class of truths, i. e., there may be heterical extensions but the heteras are inter se similia; there may be heterical times and heterical wheres, but inter se times are similia, and so of wheres, and therefore, each makes but one class.

But again, if we take an aggregate existence, a piece of iron for instance, and move it to another place, we will perceive that it is not now in the same WHERE in which it was before it was moved, it has changed its place in space. And hence the HETERATION of WHEREs occupied at different times by one and the same existence, gives us the knowledge of that existence's motion. While the same points in an existence remain in the same WHEREs, no discrimination of any points WHEREs, of course, can be made, and without the HETERATION of one and the same point's WHEREs, no motion of that point can take place. This truth of motion, again, forms of itself a class of truths.

But again: we have in our minds testimonial truths. And testimonial truths are those, which we receive upon the testimony of others without bringing them up from FACTS for ourselves. And every witness must testify to that only which has come under his own observation, or to a truth which his own mind has wrought out: or, if a person state that which has been told to him by another, and the other but related what he had heard, in order that there may be any truth at all in the story, there must have been some person, whose mind brought the truth in question up from FACTS. For some truths, we are entirely dependent upon the testimony of others: as that Caesar was assassinated, Columbus discovered America, etc., while there are others, which we may gain for ourselves from nature and also receive them

from testimony: as that the sun and moon shine upon China. And respecting those truths, which are conveyed to our minds by the testimony of others, it is to be observed that there must always be some analogy in the whole or in the parts, between a truth related to us and some truth of which we already have the knowledge: otherwise we can gain no knowledge by such relation, should there be no analogy existing between the truth, which a friend desires to relate to us, and some truth with which we are already familiar, no conception of the truth in his mind can be established by words in our own. The king of Siam is said to have laughed when told that water, a fluid, would congeal and become ice, a solid: but if he had had already no knowledge of a solid or fluid, he would have had nothing at which to laugh: for he could have known nothing about the subject of the conversation. If a traveler should discover in some unexplored country an animal with feet like those of a cow, a body like that of a lizzard, and a head like that of a crane, by using these things with which we are familiar to explain the appearance of the various parts of this newly discovered creature, he could give us a conception of his animal as a whole. But should a traveler discover an animal, which in the whole and in the parts, was entirely unlike anything of which we have any knowledge, he could not possibly, by language, give us any conception of what he had seen. And in order that we might gain any knowledge of such an animal, we would have to see the animal itself, or have a picture or sculptured image of it presented to us.

But again: we have the knowledge of existences of the imagination. These existences are peculiar and require some consideration here. Centaurs, Sphihx, Harpies, Hydras, etc., are represented to us, while these creatures really have had no objective existence in nature. Yet the mind per se has no power to create from nothing existences of any kind; even the baseless fabric of dreams is not the creation of the mind from nothing. But if existences of the imagination have no real objective existence, and if the mind can not create them from nothing, whence do they come to be subjective existences? The state of the case is this, a centaur, and all other existences of the imagination, though they have no real objective existence in nature as a combination and whole, yet all of them, partially in the parts considered, have a real objective existence. A centaur is an existence of the imagination, one part of which is like that of a man, and the other like that of a horse. Both of the parts separately considered, have a real objective existence in nature. The imagination unites these parts and from their combination creates an existence, which has a real subjective existence, but which as a whole, a unity, has no objective existence. But had the parts, separately considered, no objective existence, their unity could never have had a subjective existence. All the imagined monsters of ancient and modern times have been formed in this manner. The images in works of fiction, the Gods of Homer,

the Metamorphoses of Ovid, and the character of Hamlet and Othello are creatures of imagination, which have been collected in the same manner.

CHAPTER VII.

CAUSE AND EFFECT.

As we will have occasion in a subsequent part of this volume to treat of cause and effect, it seems necessary to prepare the way by examining the manner in which we come by the knowledge of these existences. Now we can gain no knowledge of cause except through effect. We may know arsenic as a metal; but as a poison, a CAUSE of death to animals, we can know nothing of it without first having the knowledge of the effect; that this capacious gregarium is contained in it, is found out through the effect. We can not view objects, which are potential causes, and per se determine such to be their case, a priori; it is some effect of which we first gain the knowledge, that brings to our minds the knowledge of cause. But the very instant we look upon anything as an effect, we have the knowledge of cause: for, cause and effect are but counterparts of each other. To understand, therefore, what we mean by cause, it is necessary to begin with the examination of effect.

Now an effect, in general language, is some change produced. Without change there can be no effect. If we conceive of the earth as having always existed, we can not conceive of its existence as an effect. We do not mean however, that that of which we can not conceive, can have no existence: all we mean is that we can have no knowledge of that of which we can not conceive. And if no changes whatever took place upon the earth, or in the heavens over our heads, we could never gain the knowledge of effect, and consequently we could know nothing of cause. If we consider pure space, we will see that we can not conceive of its having had a beginning, or of any changes whatever having taken place in its nature, and therefore, we can not conceive of it, as an effect. The knowledge of change must precede that of effect and cause: and when we perceive that the change has been produced by something else than the change itself, we then have the knowledge of effect and cause. We must perceive, however, that the change has been produced, or we do not come to look upon such change as an effect. Suppose the first inhabitants of earth to have looked upon the moon and to have seen her undergoing in appearance, continual changes, (and this they could not have avoided if they looked up) could they have evolved the truths of effect and cause from these phenomena alone? We think they could not. If the first changes with which men became acquainted were those of the phases of the moon, and their minds were not yet familiar with the exertion of any power in nature to produce change, providing they really believed the old and full moon to be in reality changes in the same moon, indicated by

phenomenal differentia, the comparison of these differentia would only evolve the knowledge of change. But that this change was the effect of some cause, could not be evolved from such comparison.

Now the simplest change with which we are acquainted, and which we can perceive to be produced, to be an effect, is the change of aggregate existences in space, i. e., a change of their *WHEREs*. Suppose a man should see one ivory ball strike against another and send that other some distance through space; in such case he would see a change produced, an effect. He would perceive heterical *wheres* occupied at different times by the one and same ball which was struck, and also heterical *wheres* occupied successively by the striking ball: he would also perceive that some of the heterical *wheres* of the one ball and some of those of the other, became, at different times, *homon* (Greek—neuter singular; from *homos*, a, on—the same). If we contemplate the two balls, we perceive that they are *hetera* and that their *wheres* are *hetera*; and when the striking ball moves towards the other its course is made up of *wheres* which are *inter se hetera* until it strikes, when the ball struck makes heterical *wheres*. But so soon as the first ball strikes the second one, some of the first one's *wheres* and some of the second one's *wheres* become *homon*, and from the impenetrability of matter, this could not be the case without the second one having vacated those *wheres*. In this case the change in space of the second ball is seen to be an effect, and the cause is easily perceived. The first ball commenced to move towards the second one until it touched it, and had it proceeded no further, no effect would have been produced upon the second one: but if it go on further, some of its *wheres* and some of those of the second must become *homon*, i. e., the *wheres* of the second ball at one time and the *wheres* of the first ball at another time, are in space, *homon*. Now one instance of change involving such relations, if contemplated, would give us the knowledge of effect and cause.

But again, if we tie one end of a string to a permanent object and attach the other end to the one end of a lever, every point in that string will occupy a *WHERE*, and the *wheres* of all the points *inter se* be *hetera*. The end of the lever to which the string is attached will also have a *where*, which, in reference to any point in the string, will be *heteron*. If now the string contract, some of the points in the string will take the *wheres* of other points, and some of the *wheres* of the end of the lever, and some of the *wheres* of points in the string, which were at first *hetera*, now become *homon*. And hence we see that in all those changes of aggregate existences in space, which we regard as effects and whose causes we understand, we find heterical existences with heterical *wheres*, and some of the *wheres* of one and of another becoming *homon*. Change of objects in space is also produced by what is called attraction and repulsion, but what are the causes and *modus operandi*

in these changes, philosophers have not yet sufficiently explained to us. The conversion of hetera into homon among wheres, is the *modus operandi* in those changes of objects in space, which we fully understand. Take a piece of iron and keep it all the time for a certain period under your eye, and during this period move it with your hand from one place to another. In this case we perceive that the existence moved (the iron) remains one and the same; but its wheres successively and the times of occupying them can be discriminated, and so also respecting your hand. But some of the wheres of the iron and some of the hand's wheres can not be discriminated, they are homon, though the times of occupying them by each successively are never homon always but hetera.

But again, we sometimes see one existence acting upon another, and a constitutional change following such action. Take a hammer and with it strike a grain of corn placed upon a rock, and we will see that a constitutional change takes place in the corn. This change too, we could scarcely avoid regarding as an effect the first time that we should witness the occurrence. And in this case, it will be perceived that two heterical existences come in contact and that some of the wheres of the one and some of the other become homon: and further that some of the heterical wheres of the particles in the grain of corn become homon, and hence the constitutional change. The grain of corn possessed rigidity but this gregarium was destroyed by reducing heterical wheres of heterical particles to homon. On the contrary ignite gunpowder and heterical particles immediately take heterical WHEREs.

Again, if we take a piece of ice in our hand and it melt and become water, here is a constitutional change; ice, an aggregate existence, has changed some of its gregaria and become water: and in changing these gregaria, heterical wheres of particles became homon. And in this case we must perceive that the where of the two aggregate existences, ice and water, remains the same for both; but the existences possess gregaria inter se differentia, and the times of occupying the same where are hetera. And hence, when there is during a certain period of time but one WHERE for two differential existences, the one must have occupied that where for a part of that period and then become the other existence. And when we conceive such to have been the case, we can not help conceiving of a constitutional change having taken place. Now in change times can always be heterated and when we can go a step further and heterate wheres, the change is that of place. But when we can go still further and perceive that an aggregate existence has lost some of its gregaria and taken others, which with reference to the first are differentia, the change is constitutional. A piece of iron when heated possesses different gregaria from those which it has when cold: this is owing

to a constitutional change. It, however, cools again and assumes its former gregaria.

But again, if we take grains of white sand and consider them all together in a pile, we shall have a homogenous aggregate existence, i. e., an existence in which all the particles are inter se similia: and consequently the wheres of all the particles can be heterated but the particles themselves can not be differentiated, if now we mix red sand with the pile, we then find in it particles which are not only hetera but also differentia. The pile now, therefore, compared with what it was shows change. This change, however, is owing entirely to the change in space of the particles of red and white sand.

But again, suppose we take an aggregate existence in which all of the particles are similia, so far as we can perceive, but by subjecting it to a certain process we find that particles which we regarded as similia have become plainly differentia, which is always the case in the analysis of compounds, here is to us a change of a different kind from any of the former.

And again, suppose we take two aggregate existences, in each of which the particles inter se are similia, but the particles of the one and those of the other are inter se differentia, and we put these two aggregate existences together and find that all the particles of each existence now, if compared with what they were then, are then and now inter se differentia, but among themselves they have all become now similia: here again is a change different in kind from any of the former. This change always takes place when different elements unite and form a compound.

The following, therefore, appear to be the principal changes with which we are familiar, viz: the starting an aggregate existence in an homonical WHERE into heterical WHEREs, which is a change of place; the change of heterical wheres of particles into homonical wheres, and vice versa, which is a constitutional change of the adhesion of particles inter se, as in crushing and expansion; the conversion of similia into differentia, which is the analysis of a compound; and the conversion of differentia into similia, which is the synthesis of elements, having chemical affinity for each other. Every change which takes place among existences of the non-ego involves the principles of one or another of the above examples, excepting changes in degree. And we can readily see that one homonical existence per se can not change, but that the change of any one existence is owing in part to some other existence. Every change is dependent. And as change springs from the relations of existences, within those relations must also be the cause or power to produce change. Sodium pers se does not posses the cause of soda; nor does oxygen contain it within itself; but from the relations of sodium and oxygen spring the protoxide of soduin or soda. And we see that the knowledge of change comes into our minds by comparison: and so also does our knowl-

edge of effect and cause. And without the involution of homon and hetera, or similia and differentia, or commensura and incomensura, we can not evolve the knowledge of cause and effect.

There is a change in the appearance of the moon; there is also a change in the state of the atmosphere, by the comparison of these changes, we have hetera and differentia; but neither homon or similia. And from these things per se, i. e.; from hetera and differentia, or from homon and similia, or from hetera or homon and commensura, we cannot evolve the knowledge of cause and effect. If a rock fall from the cliff of a mountain into the valley, and about the same time the ice break loose from the shores and float down a river, here also are changes, but they do not come together anywhere, so as to bring hetera into homon, or vice versa; similia into differentia, or vice versa; commensura into incommensura, or vice versa; so that we can evolve from their comparison an effect or cause. Hetera must meet somewhere in homon or vice versa; or similia in differentia, or vice versa; or commensura in incommensura, or vice versa; in order to bring to our minds effects and causes.

Now of causes there are three classes viz: expended, acting and potential. Causa striarum of the rocks is an example of an expended cause. The floating icebergs, as believed, striated in their course the rocks. But they have vanished and ceased to be causes. The flowing of the water in the river is an effect of an acting cause, and gunpowder unexploded is an example of a potential cause.

CHAPTER VIII.

NAMES.

We come now to the consideration of names. When we reason and use words, we must necessarily see to it, that our words have some definite meaning, otherwise we will but veer about over subjects at random without making comparisons in such a manner as will evolve truths. In the most common affairs of life we reason either well or or ill, and we lead others into our trains of thought and reasoning by the use of words. So much have words to do with reasoning, that Archbishop Whately concluded logic, or the science of reasoning, to be entirely conversant about language: a mistake similar to that of supposing the symbols of Algebra to be the only things about which that science treats. But the relations of existences inter se are subject-matter of the science of reasoning and of every other science. And as words are used to designate the results of these relations, the words themselves must subjectively bear some relations to each other and to the existences which they are used to designate: and so far as they are brought by the mind to play a part in the relations of the ego to the non-ego in reason-

ing, they are the subjects of the science of reasoning. And after what has already been said in the previous chapters, we do not think it will be very difficult to understand the functions of words in the processes of reasoning.

We have already seen, that hetera lie at the very foundations of our knowledge. That which is so related to the ego, that it may be an object between which and the ego, some truth depending upon such relation may come to our knowledge, we call an existence. And words when spoken are to the ear signs of cognitions of the person speaking them; when written on paper they are signs for the eye. And when existences come to our knowledge to be existences by the power of the mind to evolve the relations among which it is placed into hetera, these heterical existences are known only as hetera, and no one of them is distinguished from another except as separate existences. And when we consider one of these heterical existences independently of its relations to others, and we wish to set out a word as the sign of our cognition, we use a name to call to the mind of the hearer or of him who sees the word written, one of hetera, without distinguishing in any manner this one from others.

And hence words, for logical purposes, may be divided into two classes, viz: names which are used by us to distinguish existences inter se, and names used to call to the mind existences without distinguishing them inter se. To the later class belong such words as existence, being, thing, entity, phenomenon, etc. These non-distinguishing names are few in number in all languages. And taking up the second class, i. e., names used by us as signs to distinguish existences inter se, we will notice those few in number which distinguish hetera inter se. Names to distinguish hetera inter se are such words as the following: this and that, these and those, once, twice, first, second, ego and non-ego, etc.

But every conscious existence has a where, which it occupies, and the relations of wheres occupied by conscious existences are expressed by prepositions. The where, however, and the conscious truth which occupies it, are differentia. And we will, perhaps, be better understood if we sub-divide that class of names, which distinguish existences inter se, into six classes, viz: names of homon, of hetera, of similia, of differentia, of commensura and of incommensura; and keeping this sub-classification in view, we will treat of them somewhat promiscuously.

Now it must be evident that sometimes a simple word is used as a name, as iron, glass, ice, etc., and sometimes names are compound words, as hydrophobia, etc. All those words, which by grammarians are distinguished as nouns, are names. Some of these are names of simple existences as red, taste sound, etc., and some are names of aggregate existences as iron, wood, coal, ship, etc. And all those words too, which are grammatically adjectives, are logically but names of the gregaria of aggregate existences. In the ex-

pression, "A red house," the word red shows that this facial gregarium is one of the gregaria of the house, and of this facial gregarium, it is the name. And all adjectives of the positive degree when joined to aggregate existences, name some one of the gregaria, facial or capacial, which along with others constitute the peculiar aggregation named by the noun to which the adjective belongs. In the expression, "A good man," the noun man is the name of an aggregate existence; and the word 'good,' which is joined with it, is the name of one of the capacial gregaria supposed to be in the aggregation. "A fusible metal," is an expression of the same kind. And it is to be remarked that those adjectives which are the names of facial gregaria may stand alone as the names of either the subject or predicate of a proposition: while names of capacial gregaria require, generally, in our language, the names of existences in which the gregaria named by them severally, are aggregated, to go along with them when they are made the subject of a proposition. We can say that white or red is a color; but we can not say that a round is on the table, and we should rather say a round thing is on the table. And when we wish to use such words, which are the names of capacial gregaria, as names by themselves in the subject of propositions, we usually change the form of the word: thus round is changed into roundness, rectangular into rectangularity, heavy into heaviness or weight, etc.

The article A or AN is continually used in logical propositions and it always has a significance. This article is the name of an heterical relation: it is derived from ANE; German EIN, and means ONE. And therefore, the expression, "A red house," contains three names viz: house, the name of an aggregate existence; red, the name of one of its gregaria, and A (one), the name of the numerical relation of the house. The article THE, is the name of an homonical relation, and it is used to distinguish homon from hetera: as, "This is the horse which we saw yesterday," "Thou art the man," etc. Sometimes the adjective SAME and also the word SELF are used along with the noun to which the article refers: as, "The same horse," "The gate itself." The articles, however, can not be used alone, either as the subject or predicate of a proposition which is concerned about anything else than names. They, however, frequently appear in propositions along with other names, and their functions, therefore, ought to be understood.

Prepositions are the names of relations among existences and among the WHEREs of existences in space: as, "The log under the bridge," "In the house," "Over the river," "Beyond the tree," etc. Adverbs are the names of relations of time and space and modes of acting: as here, there, then, now, bravely, dilligently, etc. We do not propose to treat of words any further than it is necessary to the understanding of reasoning, and we have perhaps, said enough about simple names for the present.

But frequently several words taken together make but one distinguishing name: as "A red color" is the name of a single and simple existence. Again, "Charles Carrol of Carrolton" is but one name. And "The miller who ground the grist yesterday and who died to-day" is but one name, and after it we may add, "was a man of benevolence," another name. "In the house" and "By the sea-side" are distinguishing names of heterical wheres. Such names are called by logicians **MANY WORDED NAMES**.

But again, a collective noun or name stands as a sign to distinguish an aggregation of aggregations: as, the assembly, a multitude, a battallion, regiment, etc. And when such names are used, it is usual and frequently better for the sake of perspicuity, to connect the name of an aggregate existence, which with others of the same kind make up the collective aggregation, with the collective noun: as the assembly of the people, a multitude of women, a regiment of geese, a society of prairie dogs, etc.

Again: a general or common name is one used in the first instance to distinguish an individual existence, either simple or aggregate, which has been differentiated or incommensurated from others: but each of those existences, which, with the first existence named are *similia* or *commensura*, must receive the same name, and therefore the name becomes general or common. A common name is the name of *similia* or of *commensura*. Existences *inter se similia* never receive a name other than a common one for each individual, for the simple reason that, after we have distinguished them into *hetera* there is nothing by which we can distinguish them further. We may call them 1st, 2d, 3d, etc., but such naming distinguishes them merely into *hetera*. And in order that any existence may be given a name to distinguish it from others otherwise than heterically, it and the others must be *inter se differentia*. If we take ten grains of corn *inter se similia*, and call one Alpha, another Beta, another Gamma and so on, our naming has amounted to nothing; for so soon as our eyes are turned away from them and they have changed places, we can not afterwards tell, which one is Alpha or Beta, etc. All those existences, therefore, which can not be discriminated by us further than into *hetera*, must from a mental necessity, when not numerically considered, receive from us a common name. But it may be said that **HORSE** is a common name, yet horses can be discriminated. This is true, and then they also receive distinguishing names; not indeed, to distinguish the individuals from objects, which are not horses, for the name **HORSE** has already done that, but to distinguish them *inter se*; as black horse, white horse, Arabian horse, the horse with short ears, the near horse, the off horse, etc. In like manner **COLOR** is a common name, yet colors *inter se* can be discriminated into *differentia*, which receive distinguishing names, and which names may also be common names, and they will be, if any color, discriminated from others, have *similia*.

Now if a man should place before himself a horse, a tree and a stone, by examining them, he would perceive, that the one possessed the capacial gregarium of animation; the other the capacial gregaria of vegetation, and the last, capacial gregaria of a different kind from either of the former. These three objects, therefore, would be inter se differentia: they are the three aggregate nominal truths, and we may distinguish them inter se by the names animal, vegetable and mineral. And afterwards every object possessing the capacial gregarium of animation and the horse, as aggregate nominal truths, would be similia, and therefore it must be called by the name animal. Animals, however, may be differentiated into aggregate primary propositional truths, and so on in a like manner, which, we saw was pursued with those simple existences, which we call facial gregaria grounded in the non-ego. And it must appear, that if every aggregate existence, with which we are acquainted, possessed the like number of facial and capacial gregaria, which were inter se similia, aggregate existences could only be discriminated into hetera, they would all be similia and they could have but one common name. But the facial gregaria inter se differentia are many and the differential capacial gregaria are innumerable; and could we find an aggregate existence, in which all the facial and capacial gregaria excepting one were like those of gold, yet as it differed from gold in one respect, it and gold would be differentia, and consequently it would have to receive a name to distinguish it from gold and other things. Common names, therefore, are the names of the individual existences severally, which upon one and the same generalization of existences are similia or commensura.

A proper name is the name given to a single existence to distinguish it from all others in the universe. And it must be perceived that, besides the capacial gregarium of animation, which distinguishes animals, animals are made up of various other gregaria, both facial and capacial, by which we can easily distinguish them inter se. And after that we have sub-divided them into species, we are still able to distinguish the individuals of the same species. Take for instance, the species or genus homo, and after that we have divided this species into the five races, we can easily distinguish the individuals of the same race. Nature is so fond of variety that, in the largest cities two men can seldom be found, who are in all respects similia. And this variety of gregaria outside of those upon which the generalization, in respect to which men are similia is made, enables us to impose with effect proper names upon individuals. Daniel Webster, outside of those gregaria which made him and other men inter se similia, possessed gregaria facial and capacial, by which he could be distinguished and known from others. City is a common name, and yet every city, besides the juxtaposition of houses and the jostling of men, has other relations and dissimilar plats and surround-

ings on the earth by which we may distinguish them by the proper names London, Paris, Philadelphia, etc.

Correlative names are the names of existences so related to each other that the mention of the one suggests the relation: as father and son, husband and wife, mother and child, cause and effect, king and subject, etc.

A concrete name is the name of an existence grounded in the ego and considered with reference to its ground in the ego, or of an existence grounded in the non-ego and considered with reference to its ground in the non-ego; in other words the existences (for, names in themselves can not be concrete or abstract) distinguished by what are called concrete names, have their locations in the ego or in the non-ego, assigned to them by the mind when their name are spoken, and therefore, they are concrete; and from this circumstance the names of such existences are called concrete. An abstract name is the name of an existence for which the mind assigns no location, but merely views the existence subjectively without determining its ground either in the ego or non-ego, as whiteness, fusibility, roundness, etc. The adjective names of facial and capacial gregaria, such as white, red, sweet, fusible, combustible, conscious, etc., are generally concrete; and when the existences for which they stand are to be viewed in the abstract, we change these names grammatically into nouns: as whiteness, redness, blackness, consciousness, etc. We may however use adjective names to denote abstract existences, as white is not black, i. e., whiteness is not blackness.

Names have been divided into positive and negative. This division, however, is made altogether from the combination and appearance of words, and not from the functions of words as names. The division made by Aristotle into definite and indefinite is a much better one: as definite white, red, man, horse, etc.; indefinite not-white, not red, not man, etc. Definite names, then, are names of individuals separately, or of the individuals severally of a class; and indefinite names are the names of anything not denoted by the definite name, which is always part of the word used as an indefinite name. The truth is that such names as not red, not man, nothing, non-entity, etc., can have no existence in any language independent of propositions, they spring up in propositions, and in order to understand them, we will have to treat of propositions. There is also another set of names, such as blind, mute, deaf, etc., which have been called privitives; they certainly exercise the functions of names, but we can understand them much better after having treated of propositions. It has been usual with writers on logic to treat explicitly of names and their divisions, and we have said this much by a kind of duress, although after names have been divided into names of hetera, homon, similia, differentia, commensura and incommensura, we deem the other divisions of no great importance.

NOTE.—It seems necessary at the end of this chapter to notice briefly, what we regard as erroneous in the chapter on names in the work of J. Stuart Mill on logic; not because we wish to find fault with Mr. Mill more than others, but because Mr. Mill is one of the strongest writers upon logic in the English language, and the futility of the subject is, therefore, best shown from his work. On page eighteen, of the edition published by Harper & Bros., he says, "A general name is familiarly defined, a name which is capable of being truly affirmed in the same sense of each of an indefinite number of things. An individual or singular name, is a name, which is only capable of being truly affirmed in the same sense of one thing." And again on the same page, "A general name is one which can be predicated of each individual of a multitude; a collective name can not be predicated of each separately, but only of all taken together." Now upon the foregoing, we would remark that a NAME can not be AFFIRMED of anything; for, every expressed affirmation is contained in a proposition, and that, which is affirmed in any proposition, can not be a NAME, as we will see, when we come to treat of propositions in chapters X, XI, XII, XIII, XIV and XV. Mr. Mill, in his explanation of names has all the time had in view the generally, we may say, the universally received hypothesis that, in propositions the predicate term is affirmed or denied of the subject, or that the thing denoted or connoted, to use a term of Mr. Mill and the schoolmen, by the predicate term is affirmed or denied of the thing denoted or connoted by the subject-term; a theory which we hope to be able to show hereafter to be entirely erroneous, and which has led Mr. Mill and other eminent writers into erroneous conceptions of names. But again on the same page as before, "A concrete name is a name which stands for a THING; an abstract name is a name which stands for an attribute of a thing." And hence the name of an attribute of a THING, is the name of NOTHING, unless an attribute be a THING of a THING. But on page thirty-two he tells us that, "When we have occasion for a name which shall be capable of denoting whatever exists, as contradistinguished from non-entity or nothing, there is hardly a word applicable to the purpose, which is not also, and even more familiarly taken in a sense, in which it denotes only substances. But substances are not all that exist; attributes, if such things are to be spoken of, must be said to exist: feelings also exist. Yet when we speak of an object, or of a thing, we are almost always supposed to mean a substance. There seems to be a kind of contradiction in using such an expression as that one thing is merely the attribute of another thing." From this, it seems that Mr. Mill's definitions of concrete and abstract names ought to have read: a concrete name is a name which stands for a substance; an abstract name is a name, which stands for an attribute of a substance: for, otherwise, if both substances and attributes are to be called THINGS, then a concrete name, according to Mr. Mill, covers these and leaves abstract names without an object to light upon. But Mr. Mill would scarcely agree to this change of words in his sentences; for, he tells us that, "White also is the name of a thing, or rather of things." Mr. Mill, we presume would not go so far as to call white a substance, but would consider it rather as an attribute of a substance. Yet in the next sentence he tells us that, "Whiteness, again, is the name of a quality or attribute of those things" (whites). That whiteness is the attribute of WHITE is certainly strange enough. But he would probably say that, whiteness is not the attribute of white, but of white things; for on the next page following the former he tells us, "When we say snow is white, milk is white, linen is white, we do not mean to be understood that snow, or linen, or milk is a

color. We mean that they are things having the color" (white is their attribute). "The reverse is the case with the word whiteness; what we affirm to be whiteness is not snow, but the color of snow." Well, WHITENESS then is the name of the color of snow, but such being the case what is WHITE the name of when we say snow is WHITE? It may be answered that WHITE is the name of snow itself and of all white things, as Mr. Mill has said previously. Well then, if such be the case, what is SNOW the name of? Mr. Mill's language is merely a jargon. But Mr. Mill proceeds to divide names into connotive and non-connotive, and this division he considers of the most importance; "And one of those which go deepest into the nature of language." "A non-connotive term is one which signifies a subject only, or an attribute only. A connotative term, is one which denotes a subject and implies an attribute. By a subject is here meant, anything which possesses attributes. Thus John, London, England, are names which signify a subject only. None of these names, therefore, are connotative. But white, long, virtuous, are connotative. The word WHITE denotes all white things, as snow, paper, the foam of the sea, etc., and implies, or as it was termed by the schoolmen, connotes the attribute whiteness. The word white is not predicated of the attribute, but of the subjects, snow, etc.; but when we predicate it of them, we imply, or connote that the attribute whiteness belongs to them." Now in the above sentences, the misconception of the meaning of propositions first spoken of by us, is commingled with the confusion respecting concrete and abstract names, which we noticed a moment ago. We do not wish to fill our book with strictures upon the works of others, which is apt to be regarded at best as sensorious. The best way to cure errors is to bring forward the truth and let it be examined. And we repeat the remark that all the divisions of names, after that they have been divided into names of homon, hetera, similia, differentia, commensura and incommensura, are of but small importance for the purposes of explaining the reasoning processes. These six classes lie at the foundation and are used in assisting the understanding in drawing its conclusions; the other classes are useful, if useful at all, merely for the purposes of distinctions in mentioning things, but they do not assist, but rather impede, the progress of science.

CHAPTER IX.

CLASSIFICATION OF PROPOSITIONS.

In the previous chapters, we endeavored to obtain classifications of those objects with which we are familiar, and to treat of names used to mark and distinguish truths. And it must have been observed, that what former writers have called attributes we call existences, and when these existences co-exist, we name them gregaria. Among most logicians, and especially among the schoolmen, what they call attributes are said to inhere in a substance. But of this substance in which attributes inhere, we have not been able to gain any knowledge whatever independent of the attributes. And we regard the name ATTRIBUTE as calculated to mislead, and therefore we do not it at all. And a substance stripped of gregaria is unknown to us; independent of the capacial gregaria, we know nothing of the ego, or of any mind; and stripped of facial and capacial gregaria, we know nothing of matter. And the gregaria, of which we know something directly, may with as much

propriety at least be called existences, as those things which our thoughts, from our knowledge of gregaria, lead us to suppose to be in some manner, we know not how, the causes between the ego and non-ego, of those gregaria. We are able to say with confidence that one thing per se can not be a cause, i. e., no change or effect can come out of it. We are able to say with equal confidence that red, white, sweet, etc., have not always been to us existences, but that with us they had a beginning; and therefore we conclude that our mind in and of itself must be something, and that there are other somethings, whose relations to the mind cause these existences, which we call red, sweet, etc.

Now when men were forming language, they were endeavoring to distinguish by the names, which they hit upon, certain truths which had come to their minds. But if their names do not point out clearly to our minds, well defined truths, we lay them aside and endeavor to supply their places with more suitable instruments. And it must appear evident to every one that had any person attempted to compose a treatise on logic in the infancy of language, in order to have succeeded in stating what is now known about it, he would have had to run away ahead of his generation in the knowledge of things, and to have invented and explained terms which have cost the human intellect ages of labor to furnish to us. But happily for us the laboratory of thought has been vigorously operating for many a thousand years before we have been called upon to enter the arena of mind. Instruments for stamping truths have been prepared to our hand by nations, each independent of the others. And although language always has been, and always will be behind the wants of a people who push their inquiries beyond the already occupied fields of knowledge; yet the advance usually proceeds with so gradual a pace, that there is not much difficulty usually, in forming the language chart of the newly discovered territory.

Now in the preceeding pages, we endeavored to show how we obtained and classified the truths of which we treated: we also applied the names used for distinguishing them. At the same time, therefore, that we were tracing the processes of the mind in gaining knowledge, we were also furnishing and setting down the signs by which to distinguish the knowledge obtained. And if words, as it has been said, are the forts established to guard and keep mental acquisitions, we should expect a writer, who puts his truths carefully into groups for future use, to fortify them with proper terms, as he passed along. This we have endeavored to do as well as we were able; and then we took a view of these names or forts. We must proceed, therefore, to connect those names, or forts, together and consider the results. This is done by the use of propositions.

A proposition, in general, we define to be the result of the comparison of existences made by the mind and expressed in words; and under this

general definition of proposition we make two classes of propositions viz: logical and conclusional propositions. A logical proposition is one in which the result of the comparison between two existences made immediately by the mind is expressed in words; a conclusional proposition is one in which the comparison between two or more existences is made immediately by means of a particular EXISTENCE or existences and the result of the comparison is expressed in words. The sun is an existence, fire burns, snow is white, etc., are example of the first class. In each of these propositions there is a mental comparison immediately made between two existences, and the result of the comparison is expressed in words. The expressions; the sun is and the sun is an existence, are equivalent: fire burns, is equivalent to fire is the cause of burning sensations: fire itself is the effect of chemical affinities. And hence every proposition fully stated requires a subject and predicate, i. e., a name to distinguish the truth upon which the mind first looks, and also a name to point out the truth connected with the first in comparison. The comparison may frequently, by a mode of speech, be expressed by using the name of the subject only with a verb: and in such cases the other existence compared is suggested and compared by the verb, i. e., the verb both points out and compares the predicate with the subject. This is generally the case, when the subject or first existence considered is the reputed cause of the second one: as fire burns, ice cools, the sun shines, the mind thinks, etc. This is also the case when the first existence is looked upon as the subject upon which some effect is produced: as beauty fades, water runs, leaves fall, etc. But all such propositions may be made by wording them differently to set out a subject, a predicate and a copula, i. e., in each of which propositions, two well defined truths shall appear, the one as subject and the other as predicate, with a copula to express the result of the comparison. The verb used in our language, as the copula, may always be made to be some part of the substantive verb TO BE; as snow is white.

Now respecting the meaning of this copula in propositions there has been much dispute among authors. When we say that the sun is, we mean that the sun exists, is an existence. This, indeed, is the primary meaning of the verb TO BE. But besides this meaning authors tell us that it has another; as when we say John is a man; they tell us we use the copula is merely as the sign of predication. And although in the proposition, the sun is, they tell us is is a predicate of itself, yet when a name is placed after it, it then passes its predicable quality over to that name. All this is certainly somewhat obscure. For, when we take from the verb TO BE its primary signification and call it a sign of predication, what do we mean by this expression? We mean, say our authors, that the copula affirms one thing of another. But I do not see that any more light has been thrown upon the subject by the change of phraseology. When we say that ice is frozen water, according to

this explanation, we affirm frozen water of ice, when in truth frozen water and ice are the same thing, and therefore, in truth, we affirm ITSELF of the subject. But if it be explained by saying that the copula shows that the subject possesses the predicate, or that the predicate belongs to the subject, as it is usually done, we answer that this explanation explains nothing. For, according to this doctrine, ice possesses frozen water, or frozen water belongs to ice—a mere jargon of words. But it is said “That the employment of it (the copula) as a copula does not necessarily include the affirmation of existence appears from such a proposition as this, ‘A centaur is a fiction of the poets,’ where it can not possibly be implied that a centaur exists, since the proposition itself expressly asserts, that the thing has no real existence.”—J. Stuart Mill. To this we answer, that a centaur has a real existence, nor does the proposition assert the contrary. Its existence, however, is grounded in the ego, as the proposition asserts, “A fiction of the poets.” Although modern logicians have arrived at more certain conclusions, in very many respects, yet in their expositions of propositions, they are as much at fault as the ancients. The truth is that the verb TO BE as the copula in propositions, maintains its primitive meaning in every instance, nor can it be shown to have any other in any case. We may, indeed, say that it is merely the sign of predication, but when we come to examine closely this expression, we will find it to be merely words without knowledge. Such expressions as these, snow is white, John is a man, leaves are green, etc., were brought into use before philosophy had made a beginning; they are natural, short and convenient modes of expression and explicit enough for the wants of mankind in communicating thought in a general manner; the philosophic interpretation of them, however, by writers upon logic, we regard as erroneous. But we must defer the further consideration of the copula until we come to the interpretation of propositions, when we hope to give a full and clear explanation of the whole matter; and we have merely adverted to the subject here, for the sake of order, and to put the reader on his guard against what we consider errors.

From the supposition that in all propositions there is something affirmed of the subject in certain cases, and something denied of the subject in other cases, writers have classified propositions into affirmative and negative. But this classification, in our view, is unscientific and built upon a sandy foundation. Every proposition, indeed, expresses a discourse of the mind, which may be denied or contradicted. But if we place before our mind a single existence either simple or aggregate, red for instance, as the subject of every proposition must be, we can deny nothing of that existence: if we say anything at all about it, we must make an affirmation. Take the two propositions, John is well, and, John is not well: and if we consider the one as a reply to the other, there will, indeed, be a denial; but contemplating

either one of them as independent of the other, and it contains an affirmation. And further, if this appear obscure, we may ask ourselves, whether both expressions are really propositions, and if they are, then they must have something in common: PROPOSITION must be the genus of which each is a species. If they be differentia, and yet in some generalization similia, they must have been differentiated from the higher class in which they were similia. But if we say that the one affirms something of something, and the other denies something of something, as is done, they then have nothing in common, excepting that each has a subject and a predicate, i. e., one existence before and another after the copula. But if the names of the two existences compared in propositions be set down, as may always be done, and we distinguish the one from the other by calling the one the subject and the other the predicate, this is merely a classification of the terms, and terms alone do make a proposition. The classification of terms, therefore, can not be the thing in common, which unites all propositions in a common class. But if some propositions affirm and others deny, these things (affirmation and denial) are differentia, and there is nothing left in which the propositions can agree excepting the classification of terms. In the two propositions "A pear is a fruit," and, "An apple is not a pear," we consider that there is no denial in either case, both are affirmations; though this doctrine will, no doubt, sound strange to those indoctrinated from the books upon logic. They affirm, however, results which inter se are differentia. This doctrine will be easily understood after that we have treated of the interpretation of propositions.

What we consider, therefore, the proper mode of classifying propositions is by the differentiating of the results affirmed. We defined a logical proposition to be the RESULT of a comparison made immediately by the mind between two existences expressed, or affirmed, in words. Affirmation, we consider, is the very thing in common in all propositions; but the results affirmed are differentia. And these results, we find, may be discriminated into six classes, and therefore, we make six classes of propositions, viz: homonical, heterical, similical, differential, commensural and incommensural propositions. It is not necessary that we should take up each of these classes and give them further attention here; for we are only classifying preparatory to a thorough investigation hereafter. Some things have to be merely stated at first, so that the explanation when it comes, may be understood.

Now each of the above classes might, apparently, be subclassified into simple and complex propositions. A simple proposition, then, would be one in which one subject is compared with one predicate, as "John is a boy." And a complex proposition would be one in which one and the same subject is compared with each of two or more predicates; or in which one and the same predicate is compared with each of two or more subjects; or in which

two or more subjects are compared with two or more predicates. What, however, is called a complex proposition is really a single proposition expressed and one or more others understood, as "John is good and wise," equivalent to "John is good and John is wise." Again, "John and James are good and wise," is equivalent to "John is good and John is wise and James is good and James is wise." "John is not good," is a simple proposition of a different kind, and "John is neither good nor wise," is a complex proposition of the same kind. And "All the Apostles were Jews," "All the boys in the house are barefooted," etc., are complex propositions. The classification of propositions into simple and complex, however, is not a classification of propositions, as such, but rather a division of them according to the number of propositions expressed and employed in a set of words which contain but one verb.

But again, propositions have been divided into pure and modal, as "Brutus killed Cæsar," (pure) and "Brutus killed Cæsar justly" (a modal proposition). This division of propositions is made merely from the appearance given to propositions by the wording of them, and it is not a division of propositions, as such, at all. The sentence "Brutus killed Cæsar justly," contains a result which will be exactly expressed by another set of words, as "The killing of Cæsar by Brutus was just"; a PURE proposition. The division has no foundation, whatever, in the nature of propositions, but rests entirely upon the wording of them.

But again, propositions have been divided into universal or general, as "All men are mortal"; particular, "John is mortal"; individual or singular, "A man is mortal"; and indefinite, "Some men are strong". We, however, reject these divisions, as divisions of propositions, as such. The words ALL, EVERY, SOME, etc., joined to subjects or predicates qualify them and make them a certain kind of subjects and predicates, but the affirmations is made in such propositions, just as it is, where these words are wanting. These words, therefore, qualify the results of comparisons only by their qualifying effect upon the existences compared in propositions, the manner of making the affirmation is in no way affected by them; they belong to subjects and predicates and not to the result affirmed which is the essence of propositions.

The sub-classification therefore, which we will make, is into categorical and hypothetical propositions. A categorical proposition is one in which a certain result is expressed as actually existing in the relation of existences, as RED is a color, red is not green, etc. An hypothetical proposition is one in which a certain result is, SUPPOSED to exist in the relation of existences, for the purpose of drawing some conclusion from it; as "If a sheep be a horse, (hypothetical) a lamb is a colt" (conclusion). This whole phrase would be considered hypothetical by writers upon logic. The hypothesis, however, lies in the first proposition, "If a sheep be a horse," the latter sen-

tence is not hypothetical, but a categorical conclusion, which expresses a result flowing actually from the hypothesis; but the hypothesis being false the conclusion depending upon it must be false also.

Now before leaving logical propositions, we must say a few things about subjects and predicates. Subjects may be divided into simple and aggregate. A simple subject is a single existence per se, as "Red is not gseen," here red is a simple primary propositional truth. An aggregate subject is an aggregate existence, as "Iron is hard." Here IRON is an aggregate existence made up of certain facial and capacial greparia entering into a kind of fasciculus, which gregaria are the things in fasciculo for which the subjective term stands and which it distinguishes. Predicates are divided in like manner. This is all that we need say at present respecting subjects and predicates: when we come to unravel the meanings of propositions, we will have to consider subjects and predicates more fully. And this brings us to notice logical conclusions, or conclusional propositions, about which we will say but little at present as they will be treated again hereafter.

A logical or ratiocinative conclusion, as already said, is a proposition in which the result of comparisons mediately made by means of certain existences, is expressed in words. In a logical proposition the result of the comparison made IMMEDIATELY between two existences is expressed in words; but in a conclusional proposition the result is not derived from the IMMEDIATE comparison of two existences, but mediately, as A is equal to B, C is equal to A, and therefore C is equal to B (a conclusion). In the last proposition, which is a conclusion, the comparison between C and B is not immediate, but mediate by the means of A. This distinction between logical propositions and conclusional propositions is important to the clear understanding of logic: for it is evident that a conclusion once gained may be made the premise in a subsequent syllogism, and unless we understand this distinction, we will not know how to get to the bottom of the reasoning process.

All those propositions which have been denominated modal, by writers, are conclusional propositions, as "Brutus killed Cæsar justly" is a conclusion. And much of what we have already said about logical propositions, will apply to conclusional propositions, we need not therefore, repeat it. Propositions, which are called disjunctive, also, are not logical propositions proper, but conclusions, the premises of which are often not mentioned: as "John is either a knave or a fool," is not properly a logical proposition, but a conclusion drawn from some premises, which are found in and can be made out of John's actions. What have been called hypothetico disjunctive or dilematic propositions, also, are conclusions, as we will more fully see and explain hereafter.

In this chapter we have endeavored to classify propositions so that we may be more easily understood in our subsequent inquiries. All truths, and

especially those about logic, are so interlinked that we are obliged to draw, sometimes, upon those whose explanation has not yet been given in order to accomplish the work on hand. And the subject upon which we have been engaged and which we must yet consider more closely, has been misunderstood, as we believe, by all writers heretofore upon logic.

CHAPTER X.

HOMONICAL PROPOSITIONS.

We have defined a logical proposition to be the result of a comparison between two existences made immediately by the mind and expressed in words: and a conclusional proposition to be the result of comparisons between existences made mediately and expressed in words. We will first give our attention to logical propositions. And the result expressed in every logical proposition will be either a truth or an error. If our faculties be in a perfect state and exercised in the right manner, the result will generally be a truth: but if our faculties do not act in a legitimate and sufficiently vigorous manner, we will obtain an error. In every instance, therefore, it is always necessary, in order to obtain a truth by comparison, that we should have an adequate knowledge of each of the two truths compared in logical propositions. We have already shown that all existences may be compared one with another, and that knowledge is a result brought out of the relations of existences. To show, indeed, how the mind possesses the capacity in itself to compare is no part of our undertaking; but that it actually does compare among the existences which are the subjects of its cognitions, and hence gain knowledge by the comparisons, we think, has been sufficiently shown already.

Now when the mind has gained knowledge and clothed this knowledge with words, i. e., given it as it were, a body to render it visible to others, the knowledge gained, indeed, is thus made appreciable to others, but the operations of the mind in gaining that knowledge, leave no trace behind. And did every proposition clearly exhibit the two existences compared, and also the result or truth gained by their comparison, propositions would need no interpretation, for each one would fully interpret itself. But the men who commenced language, were seeking merely for an instrument of utility in the common affairs of their lives, in which clearness of detail and precision of expression were of less importance than general availability and dispatch. And therefore, in every language, the truths which are really compared in propositions are sometimes but dimly shadowed forth, and the result of their comparison always but obscurely shown by the form of the words. And this makes it necessary, in order to obtain a thorough insight into propositions, to show what the two truths compared really are, that the result of their comparison may be clearly perceived. To this task, therefore, we now proceed;

and we will commence with the examination of homonical-propositions.

Take the proposition "Red is red," and let us endeavor to clearly set out the two things compared and the truth, which is the result of their comparison. And first, we must observe that an existence which is absolutely the same existence can not be two existences, and that one thing per se can not be compared at all: two existences must always be found in every proposition. We must also observe that when we have the knowledge of an existence, we can always make some discrimination respecting that existence: for without some discrimination we can have no knowledge. Plurality of existences is necessary to our knowledge of any one; and, therefore, absolute oneness or identity is not within our knowledge: every truth of which we have any knowledge is evolved from relations. But how then can we say that "John is John," or what is equivalent to this, "John is himself"? In order to understand this it is necessary to recollect that some truths are grounded in the non-ego and others in the ego. If we look at a tree, the relations between the tree and the ego bring to our knowledge an existence (a tree) grounded in the non-ego, and also an internal existence grounded in the ego. Now simple existences can only be discriminated by their wheres, by their times and by their effects. Many effects upon the mind are inter se similia; thus if we look at an inkstand to-day, and to-morrow look at it again; both to-day and to-morrow it will produce effects upon the mind exactly similar: yet these effects will not be the same, they will not be homon, for they can be discriminated by their times. But similar effects upon our minds can only be discriminated by their times: and where there can be no heteration of times made, there can be but one and the same existence grounded in the ego, similarity is lost in identity. And we must always recollect that by the ego, we mean my mind for me and your mind for you. For should I and a thousand other persons, at one and at the same instant of time, look at an object and be affected by it exactly alike, yet to me only one of these effects would be grounded in the ego: and all the effects upon the minds of the others in respect to myself would be grounded in the non-ego. Similar truths, therefore, grounded in the ego, which can not be differentiated, but whose times can be heterated, are not one and the same, but separate existences: they are hetera. But with respect to truths grounded in the non-ego, though their effects upon the mind may be exactly similar, or to change the form of expression, these truths may exactly resemble each other, yet if their **WHEREs** can be heterated, they are not the same but separate existences. If three men receive mental impressions exactly similar, yet any person can heterate the **WHEREs** of these effects and therefore the effects are not the same. Dissimilar truths grounded in the non-ego, or in the ego, can be discriminated into differentia, they can be differentiated; but similar truths grounded in the non-ego, whose wheres can not be heterated, are to us the same. If

we should see a rock of a particular shape and color to-day in one place, and to-morrow see a rock exactly similar in another place; the only thing which would enable us to know that these two rocks are not the same, is that their present wheres are hetera. If we should find out that the first rock was no longer in its wonted place, and we could not tell the WHERE in which it now is, we would most likely conclude the second one to be it. Respecting similar truths grounded in the ego, therefore, the heteration of their times alone destroys the identity: respecting similar truths grounded in the non-ego, time being the same, the heteration of their wheres destroys the identity. The power of the mind to heterate depends upon the time and space.

And now we look at John and receive a mental effect, and again look at him and receive a similar effect, the times of these effects can be heterated, and hence there are two similar existences grounded in the ego, which can be compared with each other. But if we project these existences and ground them in the non-ego, at the very time we last looked at John, we knew of but one where for these two subjective existences to exist objectively, and hence no heteration, objectively, of their wheres can be made; and, therefore, as they are subjectively similia, they are objectively to us homon: and hence we can say that John is John, or that John is himself. The mind can also gain a truth grounded in the non-ego and afterwards recall it by what we call memory: and as often as the mind does thus recall one and the same objective truth, so many subjective truths inter se similia, but not identical, will pass through the ego, any two of which may be compared and projected. And respecting the projection of truths from the ground of the ego into that of the non-ego, we have already seen heretofore, how existences are divided by the mind into those grounded in the ego and those grounded in the non-ego.

And hence the meaning of the proposition "John is himself," is that John, grounded in the non-ego, and HIMSELF, grounded in the non-ego are the same thing; John and John who are subjectively hetera are objectively homon. We may say that John and himself are the same thing, or that John and himself exist identically, or that John exists as himself: whatever may be the words and their syntactical relations, the two subjective existences, each of which we call John, are objectively the same, and what is affirmed by the proposition, is homon. None of these expressions, however, mark in words with entire fullness the whole of the mind's operations, but merely state or set down the existences compared and affirm the result of the comparison. And in a large class of propositions, all of that class, which we have called homonical, the result of the comparison made by the mind is homon, homon is the thing affirmed. This is always the case in those propositions which defined words, i. e., in which the meaning of a word is explained by some synonym or equivalent expression: as faithfulness is fidelity.

i. e., the meaning of the word faithfulness and that of fidelity are homon. The following propositions are similar to the one first spoken of: "Sun is the name of the orb of day;" "Death is the name of the end of life;" "Term is a name given to each of the names which distinguish the existences compared in a proposition;" and so on. All of these propositions are homonical, homon is affirmed in each one of them.

Such propositions as the one above have been called verbal, because the existences compared in them are words. And according to the old but erroneous system of predication, in such propositions, one name is predicated or affirmed of another. One name, however, can not be affirmed of another, nor can one existence be affirmed of another; the only thing that can be affirmed, in such propositions as we are now treating of, is homon. In those propositions, also, which are called real, in these, which explain the nature of the thing defined, homon is the thing affirmed; as "A triangle (the thing signified by the word) is a figure having three sides and three angles," "The eye is a physical organ by which we see," "A primary property of matter is impenetrability," and so on.

But in the proposition "John is John," which we considered a little while ago, we notice that both the subject and predicate are aggregate existences, and that each one is compared with the other in the aggregate as a totality. Now when the subject is an aggregate existence, and it is viewed as a totality, and all of its gregaria are taken collectively, the predicate must also be compared in the aggregate in all homonical propositions: for an aggregate existence, as a totality, can not be the same as a simple existence, a gregarium, and vice versa. But there are homonical propositions in which the subject, in appearance, would seem to be an aggregate existence viewed as a totality, while the predicate is very plainly a simple existence, a gregarium: we must therefore examine such propositions.

We must always keep in view that in every simple proposition, two existences and only two are compared: in logical propositions these two existences are immediately compared, and in conclusional propositions they are mediately compared. These two existences may be, each of them, simple, aggregate, or collective; yet there can but two enter into the comparison in the proposition of which the result is expressed in words. And one of the difficulties in the way of understanding propositions, is to ascertain what are really the two existences and the nature of each of them in the proposition. This difficulty has not been overcome by any writer upon logic, heretofore, with whose work we are acquainted.

Now when we say that Snow is white, or that Iron is fusible, we might believe that snow and iron, aggregate existences, are compared in totality, with their predicates respectively: this however, would be entirely erroneous. And in order to ascertain and clearly exhibit by the wording of

the proposition, the two things which are really compared, we have to state the proposition thus; One of the capacial gregaria of iron is fusibility, a proposition in which a like result is obtained as in the other, and in which two simple existences, which are the things really compared distinctly appear. And if the proposition be stated so that the homonical nature of it also shall clearly appear, it will read thus; One of the capacial gregaria of iron and fusibility are homon. And in all homonical propositions in which the subject is an aggregate existence and the predicate a simple one, it is only one of the gregaria of the aggregate existence, that is compared. In the proposition, Cataline was ambitious, when the things actually compared are clearly set out it will read One of the capacial gregaria of Cataline was ambition, i. e., one of the capacial gregaria of Cataline and ambition are homon. When we say Red is red, the result of the comparison is easily seen, because we plainly see that both subject and predicate are simple existences; but when the real subject is covered up by a term which signifies an aggregate existence, and the predicate is simple, we are misled.

And hence in such propositions as Iron is fusible, writers have said that the predicate is affirmed of the subject, or that the predicate is contained in the subject and so on, all of which expressions not only give erroneous notions of the nature of propositions in general, but per se they are utterly false: for the existence which propositionally is called the predicate is compared with the subject and the result of such comparison is what is affirmed in every proposition. And although fusibility is one of the capacial gregaria of iron, and it is contained in this aggregate existence, yet this aggregate existence in totality is not the subject of the proposition Iron is fusible, but this capacial gregarium of iron is the subject. We have already shown that in every proposition two subjective existences, i. e., existences grounded in the ego are compared: and in the proposition Iron is fusible, two fusibilities are subjectively compared, and subjectively they are similia: and then they are objectively located as homon in the aggregate existence iron, and this is the result of the comparison in the proposition Iron is fusible.

Now as there are but two classes of subjects, simple and aggregate, and so also of predicates, it would not be necessary at present to say anything further respecting homonical propositions were there not sometimes set down the words all, every, most, some, the whole of, none, both, etc., along with subjects and predicates: but homonical propositions in which these words are either expressed or understood need a further investigation. And when we say that All iron is fusible, which writers have called a universal proposition, what do we mean by the words ALL iron? As iron is an aggregate existence, let us first examine a simpler case; take the proposition All red is red, i. e., red and red are homon. Now almost any one will say that this proposition is self-evident, because were the predicate anything

else than red, it could not objectively be the same thing as the subject, which is red. Now this explanation can easily be applied to unravel the mysteries of the proposition All iron is fusible. For this proposition may be thus stated, One of the capacial gregarium of all iron and fusibility are homon. And from this proposition, it must appear, that were fusibility lacking in an aggregate existence, that existence could not be iron. Fusibility is a necessary gregarium in any aggregate existence, which we distinguish by the name, iron; and consequently it must exist in this piece, that piece, and in all pieces of similar aggregations.

The word ALL standing before iron does not indicate that the mind must have made what is usually called an induction, i. e., that the mind from a great number of instances has determined the laws of nature to be uniform, and therefore this piece and that piece will fuse. The discovery of the capacial gregarium, fusibility, in one single piece of iron, if by this gregarium we distinguish an aggregate existence from others, and mark the distinction by the word iron, will enable us to say with certainty and truth that All iron is fusible; for in doing so, we merely state that one of the necessary gregaria of an aggregate existence, which we distinguish by the name iron, and fusibility are homon. That there may be other gregaria in the aggregation, of which as yet we know nothing, does not change the case at all.

Suppose a person to be taken into a large room in which there were four kinds of balls upon different shelves around the apartment, and he be required to give distinguishing names, which would enable him to speak afterwards about the balls, respecting merely their tastes and colors. He would take up the first one at hand, and perceive that it was of a red color and had a sweet taste, and therefore he would name this ball A. Then every ball in the room that was red and sweet, as balls of colors and tastes, which are inter se similia, can not be differentiated, must be called A from a mental necessity. And by the name A, they are afterwards distinguished from those that are blue and sour, which might be called B, and from those which are white and bitter, which might be called C, and so on. But so soon as he had given the name A to distinguish the first ball of a red color and sweet taste from others, all balls of a red color and sweet taste must be called A, and if so, could he not immediately after naming the first ball, have said with perfect certainty and truth that all A is red and all A is sweet? And if afterwards, a red ball should be found that was sour, it would not be an A, but it must be called by some other name.

But an Indian, before the discovery of America, might have said that all men are red, for he had never seen any man of a different color, yet his assertion would not have been true. The ancients also, might have said and did say, that all swans are white, yet such is not the case. And the error in both these cases lies in taking the gregarium of a particular object or objects

and making this gregarium in our mind, one of the necessary gregaria to distinguish this object from others, when it is not so: there were other things red besides Indians, and other things white besides swan's, when animals were distinguished by names: the color was not one of the gregaria by which these objects were necessarily distinguished.

But we have said that aggregate existences are distinguished inter se by the facial and capacial gregaria co-existing. And hence did one aggregate existence contain similar facial but not similar capacial gregaria with another, the two aggregations would not be similia, and they could not be intelligently distinguished by the same name. A distinguishing name is a word taken at pleasure to distinguish existences inter se; and when it stands for an aggregation, any one of the gregaria sine qua non, can not be lacking, and the aggregation be called by the same name as an object in which it exists. Charcoal and the diamond are said to be, as elements, similia, yet the gregaria differ and consequently we can not speak of each intelligently and use the same name.

But how then, say you, is it that a black swan and a white one may both be called swans? Simply because they are differentiated into swan's irrespective of their colors, just as red and white, as we have seen, are first differentiated into color, and then distinguished inter se, by the names red and white. All men are mortal, is a proposition of the same kind as All iron is fusible. Mortality is one of the capacial gregaria sine qua non of man, and a living being not subject to death would not be a man. The proposition, All men are mortal, however is a very different one from, All men are mortals; the first affirms homon of mortality and one of the capacial gregaria sine qua non of man; the second affirms man and one of the aggregate existences called mortals to be homon. All men are animals, and, All sheep are animals, are similar propositions, and they may be thus interpreted: man and one species of animals are homon, sheep and one species of animals are homon.

But to pursue further the effect of the word ALL in propositions, if when man was first placed upon the earth, he had lived to the age of ten thousand years without a death occurring, and if during that period he had invented language and distinguished himself by the name man, it is plain that mortality would not have been in his mind one of the capacial gregaria of himself: he would not at least have known this by direct observation. And if during this time, no constitutional changes among external objects had come to his knowledge, it is evident that he would have known nothing at all about the capacial gregaria of objects; but all the names in his language would have been signs to distinguish simple existences inter se, and aggregations of facial gregaria. And therefore all the aggregate existences now classified by their capacial gregaria and marked by distinguish-

ing names, would have remained unclassified. And then each one of the facial gregaria, which was a *sine qua non* of any class, would have been a necessity in order that any object might have been called by the name given to individuals of the class. Names, of course, under the circumstances would have been few in number. But suppose now, that at the end of the period above spoken of, one of the human species had died, here would have been to mankind a new truth learned by observation. And were this instance of death then made known to all the living, all subsequent deaths would not have been new truths, but other instances of similar truths. And although *non simile est idem* or *non similia sunt idem*, objectively, yet subjectively *similia* are the same thing if time be left out of the question. And hence respecting the knowledge of truths in the mind, the recurrence of *similia* are regarded and often spoken of as other instances of the same truth, although they are not *homon* but *similia*; their times are *hetera* and therefore the truths are *similia*, but were their times *homon*, the truths also, would be subjectively *homon*. Now if we have gained the knowledge of one individual of *similia*, we have gained all the knowledge we will ever have of the *similia*, excepting their number or instances. And therefore after one death had occurred, the question would have been, men being *similia* in those gregaria which together make the object distinguished by the name man, is death one of these *capacial gregaria*? That it is could have been proved to men under the above circumstances only by a process of reasoning which we shall develop hereafter. (See book 1, chapt. xxii.) But so soon as it is established to be such, it is a *sine qua non* of man and hence we say that death and one of the *capacial gregaria* of all men are *homon*. And as aggregate existences are composed of certain facial and *capacial gregaria*, which are the very things which distinguish them into *elasses* of *similia*, when any one of these gregaria *sine qua non* is known and given a name, it may be made the predicate of an *homonical* proposition, in which the word *ALL* names the *sum totum* of the aggregate existences for any one of which the noun placed after *ALL* stands as the name. And hence that all iron is fusible, when fusibility is once in our minds a *sine qua non* of iron, is a necessity of our minds. It may be said that fusibility is not a *gregarium sine qua non* to distinguish iron from other things; for gold and other metals possess it. This is true; but go one step back into the class of things called by the name metal, and we will find fusibility to be one of the distinguishing gregaria, and in subclassifications this *gregarium* must pass into each of the subclasses; for they, each of them, under the name metal possessed it. And hence by adding the words *ALL* and *EVERY* to the name of an aggregate existence and then making the *TERM* the subjective one of an *homonical* proposition with a simple existence as the predicate, we show this simple existence named in the predicate to be one of the gregaria *sine qua non* of the aggregate existence named in the subject.

All gold is proof against the effect of nitric acid, i. e., one of the capacial gregaria sine qua non of gold, and proof against the effect of nitric acid are homon.

But we must now examine the function of the word *SOME* when placed before the name of an aggregate existence in a proposition. Take the proposition *Some ink is red*, i. e., one of the facial gregaria of *some ink* and *RED* are homon. Now it must appear that the facial gregarium here mentioned is not a sine qua non of ink, but that it is one which compared with some other color, enables us to differentiate inks. *SOME* therefore, as it names the part of a whole, shows also by being placed before an aggregate existence in homonical propositions, that the gregarium, which appears as a simple existence in the predicate, is not a sine qua non of the class of aggregate existences distinguished by the name which appears in the subject and named by the noun after *SOME*.

We do not deem it necessary to pursue the subject of homonical propositions further at present. If the reader will carefully study what has been said already, we think he will be able to follow and understand the arguments, which we will advance hereafter. We will, however, set down several homonical propositions in the language that is used in common discourse, and the reader can change the phraseology, so as to make the result affirmed appear plainly to be homon: *Some men are black-eyed; All fowls lay eggs; All gold is maleable; God is love; An apple is an apple; A straight line is the shortest distance between two points in space; Ice is frozen water; Schuylkill is the name of a river in Pennsylvania; Washington died at Mount Vernon; We are living in the nineteenth century of the Christian era; Columbus discovered America A. D. 1492; Shakespeare was a dramatic author; Sophocles wrote *Ædipus Tyrannus*; Newton discovered the universal law of gravitation.*

CHAPTER XI.

HETERICAL PROPOSITIONS.

Having treated of homonical propositions, we hope, with partial success, we come now to speak of the second class, which we have called heterical propositions. And heterical propositions affirm results, which are directly the opposite of those affirmed by homonical ones, and consequently the two classes are differentia; and when a proposition of the one class is spoken with reference to the other, it denies the affirmation made by the other. If any person affirm that *A is B*, i. e., that *A* and *B* are homon, and another person reply that *A is not B*, i. e., that *A* and *B* are hetera, the latter makes an affirmation contradictory of the affirmation of the former and vice versa.

Now if we take two twenty dollar gold pieces which are inter se

similia, and lay them before us, any person will say this piece is not that one. But the two pieces being inter se similia, if you hand one of them to a person, and then take it again and put the two together, and ask the person which one he had in his hand, he can not tell. How then does any one know that this piece is not that one, i. e., that the two pieces are not homon, but hetera? Simply because the WHEREAS of the two pieces at the same time can be heterated. But is not the proposition, This piece is not that one, an independent proposition, i. e., a proposition expressed without reference to any other? If it is such, then it can not contain a denial or negation of the subject, as it is generally supposed, but it positively affirms this piece and that piece to be hetera. You can not numerically count pieces of money without heterating them, and you can not express in words the heteration of them without using an heterical proposition or propositions. What is the difference between These two pieces are separate existences, and This piece is not that one; leaving the wording out of the consideration? The difference is this, the former proposition never could have been put into words at all, without the latter one having first been mentally at least enuntiated: the latter proposition must preceed the former in the mind, or a knowledge of the former never could be gained: in effect, however, the two are alike. The former proposition may be resolved into This piece is an existence and that piece is an existence and the whole expression is exquivalent to This piece is not that piece, i. e., this piece and that piece are hetera. And every heterical proposition may, in effect, be exactly expressed by the use of two homonical ones, by placing the distinguishing names of hetera, THIS and THAT, before their subjects: two homonical propositions may also be condensed into one similical or commensural one; or they may be differentiated or incommensurated, in differential or incommensural propositions, as we shall see hereafter. But there must be an heteration of existences in the mind before any proposition whatever can be expressed; for we have already shown that the process of heteration lies at the very foundation of knowledge. And this process of heteration can not be a negative process; it must be positive or it would amount to nothing, and its positive character can not be expressed but by an affirmation. This has been overlooked, heretofore, by all writers upon logic. Because the particle NOT is found in the proposition, it has been universally believed that the predicate denied something of the subject, or that the predicate was denied of the subject; a proposition, which follows legitimately enough from an other, which is that when this particle is omitted, something is affirmed of the subject, but both of these suppositions are untrue. The predicate is no more affirmed or denied of the subject in propositions than the subject is of the predicate; the two existences are compared, the one with the other, and that which is affirmed, in all cases, is the result of the comparison. It is impossible for the human mind to affirm or deny one

existence of another; all that we can do is to affirm some relation existing between existences.

One and the same existence of the non-ego can not sustain heterical, similical or differential relation to the ego in an homonical time; for if it could, we could have no knowledge of identity. When we say, therefore, that A is not B, we do not mean that A does not exist, or that B does not exist, for both must have an existence grounded in the ego at least, or we could not put their separate names down on paper; but, by A is not B, we mean that A and B exist heterically, that A and B are hetera. The particle, NOT, therefore, in propositions, stands as the sign of heteration made by the mind, but the result of the heteration is positive, and it is affirmed in all propositions containing this particle. And we lay down this rule: That whenever the wheres of existences grounded in the non-ego can be heterated in an homonical time, and whenever the times of existences grounded in the ego can be heterated, the heterical relations of these existences are expressed in heterical propositions.

In homonical propositions we saw that the wheres of the two existences compared, could not at the same time be heterated. When we say, John is John, the subject and predicate subjectively have the same where, but not an homonical time: John and John objectively have the same where at the same time, and therefore, objectively they are homon. But the objective John and the subjective John are hetera because their wheres at the same time can be heterated; and John and John are subjectively hetera because, though their wheres are homon, they can not have an homonical time. And, therefore, homonical and heterical propositions contradict each other, when their subjects are similia in every respect, and their predicates similia leaving the particle NOT out of the consideration.

Now in heterical propositions, we make no account of the similarity or dissimilarity of existences; all we care about, is to be able to heterate the wheres of existences grounded in the non-ego at any given time, and the times of existences grounded in the ego, and then we affirm hetera. And hence if we place two white marbles before us, the color of the one and that of the other being perfectly similia, yet we say that the color of the one is not that of the other, i. e., the color of the one and that of the other are hetera; for the wheres of these colors can be heterated. When, however, we look at A (one) marble and say The color of this marble is white, or to use the short expression, This marble is white; the color of the marble and WHITE subjectively have the same where, but heterical times; but when we project these subjectively heterical colors which are inter se similia, into the objective marble, they both have the same where at the same time and therefore, we affirm homon.

Now we have, heretofore, divided subjects and predicates into two

classes, simple and aggregate. And of simple existences, some become the gregaria of aggregations, others do not. Time and space are never gregaria. And we must have observed that it is the relations of simple existences or of aggregations in time and space, that enable us to affirm homon or hetera; the power of the mind to heterate depends upon time and space. When we say that this apple is not that one, we apparently compare one apple with the other immediately: the existences, however, which are immediately compared, are the wheres of the one and the other at the same time. But when we say subjectively, An onion is not a peach, this proposition is more than heterical and it belongs to the differential class, which we will treat of hereafter. If, however, we say this peach is not that onion, we heterate the wheres and affirm hetera, and this is shown by the words **THIS** and **THAT**. And if the reader will bear in mind, that whenever he can heterate the wheres of existences at the same time, or subjectively heterate the times of subjective existences, the proposition may be heterical, we think he will be able to detect heterical propositions, whenever he may find them in books or conversation, by some words which distinguish hetera.

We will set down a few heterical propositions for practice: Philadelphia is not New York; The Pacific Ocean is not the Atlantic; My hat does not lie on the floor; The birth-place of Washington was not Boston; This land is not that one.

CHAPTER XII.

SIMILICAL PROPOSITIONS.

When treating of homonical propositions, we showed that absolute identity makes no part of our knowledge; that in all homonical propositions, the existences compared are always subjectively hetera; that heterical results in the order of time always precede our knowledge of identity, and are the very first results obtained; that the knowledge of the existence of any simple existence is dependent upon hetera; and that unless heterical results can be obtained, chaos reigns supreme. If I see a horse to-day and to-morrow see the same horse again, nevertheless, subjectively, I have seen two distinct horses; and when viewed as existences grounded in the ego, I distinguish them by heterating their times, but when projected onto the ground of the non-ego, the heteration of their times does not distinguish them and as I can not heterate their wheres at the same time, I can not distinguish them at all, but pronounce them to be homon.

But suppose that subjectively I consider heterical existences and can not further discriminate them, and objectively also I heterate the existences but can distinguish them no further, then we call the existences similia. And hence when we can heterate subjective existences, but can proceed no further, the existences are subjectively similia, and when we can heterate objective

existences but can distinguish them no further, the existences are objectively similia. And, therefore, objective homon is always subjective similia, but not always vice versa; for subjective similia may also be objective similia. Subjective homon can not be expressed in a proposition, i. e., two acts, feelings or states of mind can not be one and the same, they must be hetera, and one thing per se can not be compared.

Take the proposition This orange tastes like that one, i. e., the tastes of this one and of that one are similia. Now the sensations of the taste of the one and of the other, as existences grounded in the ego, are similia, and when projected onto the ground of the non-ego, each one is a gregarium of heterical objects whose wheres can be heterated, and therefore, objectively, the tastes are similia. We need not proceed further at present with similical propositions. We will subjoin a few examples for practice: This apple tastes like that one; John is like his father; Time is like a silent river.

CHAPTER XIII.

DIFFERENTIAL PROPOSITIONS.

We proceed now to the consideration of the fourth class of propositions, namely, differential propositions. And when two subjective existences can be discriminated by anything besides their times, the existences are subjectively differentia. The effect produced upon and within the mind by RED is different in kind from the effect produced by GREEN, and hence the two effects are not only hetera subjectively, but also differentia. And existences, which are subjectively differentia, must necessarily, if each have a corresponding objective existence, be also objectively differentia. But how or why it is that the mind is able to discriminate between red and green, subjectively, we do not sufficiently understand. The two objects, which produce severally these different effects upon our minds. Sustain in some manner different relation to the ego: they are other different elementary principles, or the one is composed of more or differently arranged gregaria than the other. Let this be as it may, for logical purposes it makes no difference to us; every person will distinguish subjectively and objectively RED from GREEN, and consider them to be things differing in kind—differentia.

We have already stated that subjects and predicates of propositions are either simple or aggregate existences. And when both subject and predicate are simple existences, the differentiation clearly appears. That red is not green; will easily be seen to be a differential proposition. The sign NOT does not indeed of itself indicate whether the existences have been differentiated or merely heterated. But heteration can easily be distinguished from differentiation, if we look at the terms of the proposition. In the heterical proposition, This red is not that green; we see that the terms are particular names, the names of individual existences, and that the distinguishing heteri-

heterical names, **THIS** and **THAT** are joined with the common names, **RED** and **GREEN**; and thus making red and green the names of particular individuals: While in the differential proposition Red is not green, red and green are unlimited common names. The name red stands for this red, that red and for any red, and so also with green; but when we say this red, or this or that green we mean an individual. And hence in heterical propositions, the terms are individual names, while in differential propositions, they are unlimited common names. And we may assert with truth that all **RED** is not **GREEN**: though this proposition, from the custom of our way of speaking, seems to imply that some **RED** is green, and to avoid the effects of language upon our minds it is usual and better to say that no red is green. We are accustomed to say with truth that all men are not black, i. e., one of the *gregaria sine qua non* of man is not black, i. e., black and each of the *gregaria sine qua non* of man are differentia, and therefore, by implication we affirm that some men are, or may, be black. And hence the custom of language, when we say that all red is not green, would lead us to infer that we meant, some red is green, i. e., that some red and green are homon. In every proposition, therefore, in which the particle **NOT** occurs, and the subject and predicate are simple existences, if the terms are unlimited common names, the proposition is differential; if they are particular names the proposition is heterical; as John is not Charles. And this is also the case when both the subject and predicate are aggregate existences, as a man is not a horse, is a differential proposition; This man is not THAT dog, heterical. When, however, the subject is an aggregate existence and the predicate a simple one. Some further explanation seems necessary. Take the proposition snow is not black. This proposition may be thus stated: Each *gregarium* of snow and black are differentia. No snow is black means the same thing, and guarding against the custom of language, we may say that all snow is not black; better—No snow is black. All these propositions mean the same thing. All snow is white means that one of the *gregaria sine qua non* of snow and **WHITE** are homon; but no snow is black, means that each *gregarium* of snow and black are differentia. And hence in all propositions **NO** is always the sign of differentiation.

NONE is equivalent to **NO ONE**, of which words it is compounded; and when we say that none of the horses are gray we mean that **NO ONE** of the horses is gray, i. e., gray and the color of any one of the horses are differentia. No one of the horses is gray, however, is a very different proposition in its terms than **NO** horse is gray, i. e., each *gregarium* of any horse and gray are differentia.

We here subjoin a few examples for practice: A river is not an ocean; an Indian is not a negro; an apple is not a peach; no fish is a bird; a gosling is not a chicken; gunpowder is not saltpeter; steam is not water; none of the pupils are learned; a true christian is not vicious; cotton is not wool;

iron is not explosive; day is not night; cause is not effect; no horse is a stone; the rainbow is not a cloud; no color is a sound; the rocks are not trees, &c.

CHAPTER XIV.

COMMENSURAL PROPOSITIONS.

Having treated of the first four classes of propositions, we come now to commensural propositions. It must be evident to any one that if we take two simple existences which are inter se differentia, white and green for instance, we can not truthfully say that they are in any respect related to each other, except as colors; indirectly, as colors, they are similia, but directly, they are inter se differentia. Between two such existences, therefore, no comparison can be made by which a result other than an heterical or differential one can be attained. They are not similia and therefore we can not by their comparison obtain a similical result; nor from their comparison can we obtain homon. After having obtained therefore, the results, homon, hetera, similia and differentia, in order to obtain propositions, which will render results different from those just mentioned, we must measure inter se results already obtained. But homon can not be measured, for it is an identical thing, and a thing to be measured must be measured by some other thing. But hetera, as hetera, can not be measured, for in measurement there must be some coincidence and not mere separation, and differentia, as differentia, can not be measured, for they can have nothing in common which is measurable.

Similia, therefore, are the only results, which admit of comparative measurement. We can say that this red is as red as that red, i. e., this red and that red are commensural, and if we compare one stick with another we can say that this stick is as long as that one i. e., the lengths of the two sticks are commensural, and thus we can compare many of the similia of nature and obtain commensural results. We do not deem it necessary to enlarge upon the subject of commensural propositions, as we concluded that they will be easily understood, and they will also be illustrated along with the others hereafter. We must here observe, however, that homon is at the bottom of them. When we say, this red is as red as that red, the AS RED and THAT red are homon, and by stating two homonical propositions with the word AS between them, we will readily see, how two homonical propositions merge into one commensural one: Thus, this red is red, as, that red is red. In the first proposition, the subject and predicate are objectionally homon, and so also with the second proposition, and the word AS shows that the two are commensural. We will subjoin a few examples for practice: The day was as dark as night; this candle shines as bright as that one; she looks as fresh as the rose; it is just as sweet as honey. $x+y=z$.

CHAPTER XV.

INCOMMENSURAL PROPOSITIONS.

We come now to the consideration of incommensurable propositions, the last class of logical propositions. And in incommensural propositions, the existences compared are similia in kind, but they differ in degree or quantity. When we say that this candle shines brighter than that one, we mean that there is an excess of brightness in the one compared with the other. The two are not differentia, as white and black are, but there is a difference, an excess, in the one over and above the brightness which exists in the other. The difference in the specific gravity of bodies is expressed in incommensural propositions, as gold is heavier than iron, i. e., the specific gravity of gold and that of iron are incommensural. This excess in one of the existences compared is some times shown by the use of an adjective name in the comparative degree. There are, however, three ways of expressing the excess in words, viz., A is larger than B, B is less than A, and B is not so large, or not as large as A.

Now when we say that snow is white i. e., one of the facial gregaria of snow and white are homon, we locate the gregarium, white among the other gregaria, which make up snow, so when we say that ice is colder than water, we locate the existence, which would be named by the adjective name in the positive degree in the subject ice, and by adding ER or MORE to this adjective name, and thus marking an excess, we locate also the excess in the subject. Take first the case of the comparison of simple existences, this red is reder than that red. Now leaving ER off of the adjective name and we will have BEFORE THEM, this red is red, an homonical proposition. And in the proposition this red is reder than that red, we retain the homonical red—the predicate of the homonical proposition, and add, ER to its name to snow an excess above the red which follows after THAN, and which is the predicate of the incommensural proposition. But as the predicate of the homonical proposition, was located objectively in the subject of the proposition, i. e., it and the subject were found to be homon, so the excess added to it in the incommensural proposition must be located with it in the subject of the incommensural proposition.

In the incommensural proposition, this red is less red than that red, however, the decrement is located in the subject and consequently the excess is in the predicate. And in the proposition, this red is not so red as that red, the SO RED and THAT RED are homon, i. e., the degrees of red subjectively commensural are objectively homon in the predicate of the incommensural proposition, and the particle NOT shows that the degrees in the subject and those in the homonical predicate are incommensural. In the commensural proposition, this red is as red as that red, the last two REDS, which are homon

in the predicate, and the subject are commensural, but if we insert NOT we will have; this red is NOT as red as that red, in which the last two REDS are homon, and their degrees and those of the subject are incommensural, the difference or excess being in the predicate.

Now when the subject is an aggregate existence and it is compared apparently with an aggregate existence in the predicate, in commensural and incommensural propositions, it is always one of the gregaria of each that is compared, and these gregaria compared are always similia in kind, but commensural or incommensural in degree. In the proposition snow is whiter than chalk the facial gregarium, white exists in each of the aggregate existences, but the degrees of white in the one and in the other are compared and found to be incommensura. And when we say all snow is whiter than chalk, it is one of the gregaria sine qua non of snow that enters into the incommensural proposition. And if we could say in truth that all snow is whiter than all or any chalk, the degrees *ne plus ultra* of chalk would be compared.

Before passing on to the next chapter, we must examine such propositions as; John is the strongest man in the house. This proposition at first sight would appear to belong to a seventh class of propositions, but on examination, it will be found to be merely an homonical proposition collected into a conclusion from several incommensural ones, and it may be thus stated, the strongest man among the men in the house and John are homon. And so also, Sampson was the strongest man of whom we have read, is an homonical proposition. Hercules was stronger than Sampson, is an incommensural proposition. And all propositions, in which there are superlative names, are homonical. We give the following examples for practice: Winter is colder than summer; the elephant is more intelligent than the ass; dogs are more faithful than cats; cows are more useful than rabbits; the bite of a rattlesnake is more dangerous to man than the sting of the wasp; Honey is sweeter than sugar; the note of the nightengale is more pleasant than that of the crow: $x+y < z$.

CHAPTER XVI.

PROPOSITIONS PROMISCUOUSLY.

Having gone through with the six classes of propositions, we should next in order consider their subdivisions into categorical and hypothetical; we do not deem it necessary, however, to do more than mention these subdivisions. Every one will see for himself that any proposition of either of the foregoing classes may be stated categorically, i. e., the result be affirmed as actually existing, or a result may be supposed to exist for the sake of argument. We will therefore now give some further attention to the terms and copula of propositions of all the foregoing classes.

And looking back to the nominal truths grounded in the non-ego, of

which we spoke at the beginning of our investigations, and supposing that all objects had had the same color, could we have called this nominal truth A (one) color? We have already shown that the unit is a numerical relation and that our knowledge of it is envolved from duality or plurality. And in the five nominal truths mentioned, we have hetera, from which the knowledge of FIRST, SECOND, THIRD &c., might have been evolved. But we have also shown that when numbers, the names of the individuals of hetera, or of commensural colligations of hetera, are applied to existences, and the name to distinguish individuals otherwise than heterically, is spoken or written after them, the name so spoken or written must be the name of similia, a common name. We may have a horse and a dog and the two are existences. But EXISTENCE is not a name given to distinguish existences inter se, and should we write any name, which does distinguish existences inter se after the word TWO, we will find that two will not apply unless the existences be inter se similia. Horse and dog are differentia, and their names distinguish them; neither of these names, therefore, can be written after two so as to express to us the numerical sum of a horse and a dog; as hetera existences, two may be applied to them, but not as differentia.

And respecting the nominal truths, as the are inter se differentia, two could not be joined with any name, which distinguishes them as nominal truths. But if ONE be the name of a numerical relation, as we have shown, when it is applied to a differential name, there must be more than one thing distinguished in like manner by the same name; there must be similia; otherwise the thing distinguished by such name could have no numerical relations to other things, except as hetera, which in language do not receive differential names, which afterwards become the common names of similia. And, therefore, when we say, AN EXISTENCE, by this expression we show that we have in our mind one of several or many existences, i. e., one of hetera, and when we say A dog, the expression shows that we mention ONE of similia.

Looking then at the nominal truth, COLOR, could we say that, this is A (one) color? We think not. We could say this is color, or this is an existence, and that is sound; but A color, as a name, not only distinguishes color from sound, taste, &c., but it also points out some one of similia, as colors. And hence A or AN before a name, in homonical propositions, makes them quasi similical; as this town is A Philadelphia, i. e., this town and one of the Philadelphias are homon, and in effect, this town and Philadelphia are similia. The proposition This town is Philadelphia; is an homonical proposition but the placing of A before the predicate makes the proposition though homonical still, quasi similical, there being but one Philadelphia in our mind, and THIS TOWN not being that one.

And all names excepting proper names, used as terms of propositions point out among other things, a numerical relation inter similia. In the

homonical proposition, John is John; neither of the terms point out a numerical relation; but in the homonical proposition John is A MAN; i. e., John and a (one) man are homon, the predicate term points out a numerical relation, and as it stands for the same object as John, when John is brought among the similia of which it is one, among these objects, John has a numerical relation, he is A man, one of the similia named man.

Now bringing before us again the name color, if there existed A red and A green, we would then have two colors and we could say that red is A color and also that green is a color; but upon the principle just exhibited above were there but one red object and one green object in existence, red and green would be proper names and we could not say, this is A red, or that is A green, though we could say this is a red color and that is a green color. But in the homonical proposition RED is A color; RED is brought from primary propositional truths into nominal truths, and among nominal truths, it is one of the similia, A color, i. e., RED and A (one) color are homon. But if RED be A color, how can we fully distinguish in every respect this existence from others by words, when we have it in our minds, otherwise than by calling it A RED color? And hence we see that every term of a proposition, which is made up of more than one name of simple existences, points out the results of several relations, and the numerical relation among similia pointed out by the term, is called the extension of the term.

Passing on now to the consideration of terms, which are the names of aggregate existences, take the proposition, Snow is white; i. e., a gregarium of snow and A white color are homon, and we see that WHITE is brought into and fasciculated among other gregaria in snow by an homonical proposition. Again, Snow is cold, Snow will melt, &c., are all homonical propositions, and the predicates of all these propositions are located, fasciculated in snow. We may say White is in snow; i. e., the where of a WHITE and that of snow are homon, Cold is in snow, The capacial gregarium of melting is in snow; all those gregaria co-exist in snow, i. e., a fasciculus of certain gregaria and snow are homon. And if by homonical propositions we fasciculate simple existences in an aggregate one, can we not in like manner bring together aggregate existences? When we say, The audience was intelligent, we have done so. John is intelligent, William is intelligent, Mary is intelligent, &c.; but John was one of the audience, William was one, &c.

And when a name, as the term of a proposition, stands for an aggregate existence, the gregaria taken together in fasciculo constitute what is called the comprehension of the term. And in the differential proposition, Stone is not iron, the comprehensions of the terms, stone and iron, i. e., the gregaria of the one and those of the other, are compared in fasciculo. Some of the gregaria of the one and some of the other may be similia; but if the one comprehend certain gregaria and the other certain gregaria, which

are inter se differentia, or if the one contain gregaria over and above the sum of the gregaria contained in the other, the two fasciculi are inter se differentia, and they are differentia throughout the whole extent of the similia of the one and of the other. Stone is not iron is equivalent to, All stone is not iron; better, No stone is iron; and this proposition is equivalent to No iron is stone. And hence when fasciculi of gregaria comprehended by the subject and predicate terms, are compared, the proposition may always be converted, i. e., the subject be made the predicate and the predicate the subject. This is also the case when simple existences are compared. Red is red, homonical; This is not that, That is not this, heterical; Red is not green, Green is not red, differential; This is like that, That is like this, similical; This red is as red as that, That red is as red as this, commensural; and This red is not so red as that, That red is reder than this, incommensural. But when one term is the name of an aggregate existence and the other the name of a simple existence, it is always one of the gregaria of the aggregation that is compared with the simple existence pointed out by the other term, as we have already seen: Snow is white, means that the color of snow is white, which may also be converted into WHITE is the color of snow. John is a man, may be converted into One man is John. And hence in order to ascertain the existences, which are really compared in any proposition, construct one of the terms, if necessary, so that the terms may then be transposed and give the same result as before. This may be none in all propositions.

Now prepositions as we have heretofore said, are names of relations in space among existences: thus when we say Snow is white, we mean that the color of snow is white, and snow being the name of a fasciculus gregaria, OF shows that COLOR is located as one among the gregaria in fasciculo distinguished by the word snow: it, that is the preposition OF, is the name given to the relation of color among the other gregaria in space. And hence there is no difference of affirmation between Snow is white, and The snow upon the mountain is white; the affirmations are similia. But in the latter proposition One of the gregaria of snow upon the mountain is white; in the subject we have a numerical relation (one) existing among simple existences (gregaria) located in an homonical WHERE (in snow) and named snow, which WHERE is objectively homonical with another WHERE indicated by Upon the mountain. But A (one) is not the existence compared with white; the name WHITE is differentia; one distinguishes hetera and points out the relation among similia, while the proposition is homonical; neither is any WHERE compared with WHITE, for WHERE and WHITE are differentia, while the proposition is homonical; snow is not compared with white, but the color of snow; nor is the mountain compared with white; there is nothing, therefore, in either of the foregoing propositions compared with white besides a color. All the words, therefore One of the gregaria of the snow upon the mountain;

taken together, constitute but one name to distinguish one simple existence in every respect from others, and which simple existence we compare with WHITE and pronounce them to be objectively homon.

If a, b, c, d and e be the simple existences, the gregaria of an aggregate existence, and A be the name of the fasciculated gregaria, we may then say according to the custom of language that A is a, A is b, etc. And if we wish to locate A in a some WHERE, which shall be distinguished from other WHEREs by words we may say; A upon the table is a. And if the where is not yet sufficiently distinguished, and we may say, A upon the table in the house; and still further, A upon the table in the house of John Stiles on front street between Walnut and Chestnut streets in the city of Philadelphia is A. But again, take the proposition, John's book is on the table; and we see that the subject of this proposition is a fasciculation of the subject and predicate of the proposition, This book is the property of John. We do not consider it necessary to explain the terms of propositions further: but the copula is yet to be examined.

Now in the propositions, I am; I exist, or I am an existence; we must see that the affirmation is made in the present tense, grammatically. And in the proposition, I was, Did exist, or Was an existence; the affirmation is also made at the present time, but the time of existence spoken of, is the past. I was an existence; may be rendered, The time of my existence of which I speak, and past time are homon. Columbus discovered America, A. D. 1492; may be rendered The time of the discovery of America by Columbus and A. D 1492 are homon, etc. And whatever may be the tense of the verb, the existences are always compared and the affirmation made at the present time. But respecting what is called the potential mode by grammarians, as John may be a scholar; the verb itself implies capacial gregaria; the capacial gregaria of John and those of a scholar are similia; and in the proposition, John might have been a scholar; the capacial gregaria are referred to as having existed in past time.

Now in all propositions, we say that the copula is, means EXISTS. But take the proposition, Nothing is nothing; and if is means exists, what is it that does exist, for nothing can not be an existence? But we say that it is the relation between the subject and predicate that exists. But still it will be asked the relation of what, when we say Nothing is nothing? And in order to understand this, it is necessary to consider how we came by the name nothing. Take the proposition This is nothing, i. e., This thing and no-thing are homon. Now if the subject THIS THING, means some THING and the predicate NOTHING, means no-thing, how can the two be homon? If we conceive of a witch, an old hag of a woman, with a beard, riding a broomstick, subjectively this is some thing, but objectively it is nothing, and therefore,

we can say with truth that this some thing grounded in the ego, is upon the ground of the non-ego nothing; this is nothing.

"Rom.—"Peace, peace, Mercutio, peace; Thou talketh of nothing."

"Marc.—True, I talk of dreams, which are the children of an idle brain, begot of nothing but vain fantasy."

Now had man gained the knowledge of only two objective existences and had he called the one *a*, not *a* would have been a name sufficient to distinguish the other, and he then could have said pointing to *a*, This is *a*, and pointing to the other, This is not *a*; for if it be not *a*, it must be the other existence. Both these propositions then would be homonical viz: This is *a*, this and *a* are homon; This is NOT *a*, this and NOT *a* are homon. But if he wished to show *a*, and not *a* to be differentia, or to affirm heteria in a proposition, he would have to say, *a* is not not *a*, i. e., *a* and not *a* are heteria, or *a* and not *a* are differentia. But if there were four or five existences known to man and he should distinguish one of them by the name *a*, and then pointing to another he should say, this is not *a*; not *a*, would not distinguish any one of the remaining four from the others, and consequently not *a* would be indefinite. And hence when there are two existences or but two modes of existence, and the one is named *a*, for instance, not *a* may be used as a definite name for the other, as truth, not truth—error; faculty of vision, not faculty of vision—blind; hearing, not hearing—deaf, &c. But when there are more than two existences or modes of existence, NOT, UN, IN DIS, &c., joined to names makes an indefinite name.

But suppose that we had the knowledge of but two existences, which were inter se differentia, and we should give to each of them a separate name, as RED and green for instance, we would then say red is not green. But NOT GREEN, if it be any thing, must under the supposition, be RED, and we could say, not green is red, i. e., not green and red are homon. But if we had a knowledge of three differentia, red, white and green, for instance, and we should say that red is not green, not green would stand for either red or WHITE, and would be indefinite. But what definite existence is meant in the case, by NOT GREEN, would be indicated by the subject of the proposition, RED, and hence red and THE NOT GREEN are homon. And if NOT GREEN and RED be homon in the proposition, RED IS NOT GREEN, RED and green must be differentia; for in this proposition, red and not green are homon, and in the proposition, green and not red are homon, green is, and red is; the homonical proposition, red is green, however, is not true, and the simillical proposition, red is like green is also untrue, and hence we see how homon lies at the foundation of all propositions; and we see also that the particles NOT and NO, belong always to the subject or predicate and never to the copula.

We have now gone far enough, perhaps to make our news of propositions be understood; yet it may be said that when we say snow is white and

we change the wording of this proposition into One of the gregaria of snow and white are homon, we have but changed a simple proposition into a complex one; such however is not the case. When we say Edward and John are good, we can fully express in two proposition the meaning of the above phraseology, as Edward is good and John is good. But when we say One of the gregaria of snow and white are homon, we can not resolve this into two propositions and say One of the gregaria of snow is the same thing and white is the same thing, with any sense; neither can we resolve This grain of wheat and that grain of wheat are similia, into This grain of wheat is similia and that grain of wheat is similia. And if we are right and we are understood in our views of propositions, we think it will not be difficult to explain hereafter the syllogism in all its modifications and functions.

CHAPTER XVII.

THE SINGULAR SYLLOGISM.

In every legitimate syllogism, there must be two and only two propositions, which are called the premises, and a conclusion drawn from these premises. It is also necessary that there be four subjectively heterical existences, two of which, one in each premise, must be objectively heteral, and two of which must be objectively homon, or similia or commensura inter se, and that the other two appear in the conclusion. The name of the homonical existence, or of the simillical or commensural existences, which are in the premises, but not in the conclusion, is called the middle term, because it designates or distinguishes the homonical existence or the similia, or commensura, with which, each of the existences, which appear in the conclusion has been compared, and it is by means of this homonical existence or similia or commensura designated by this that middle term, the comparison, between the other two existences is effected and the result set down in the conclusion. Now it must appear upon the principle of permutation that, if a, b and c be the terms of the premises, and we arrange a and b together and a and c together, we can have four and only four different arrangements in the premises; thus, a b, b a and a c, c a. And hence logicians have divided syllogisms into four figures, as they are called, according to the positions occupied by the middle term in the premises. This middle term may denote the subjects of both propositions, or premises, the predicates of both, or the subject of the first and the predicate of the second, or the predicate of the first and the subject of the second. And hence let a, or a and a when similia or commensura are used be the middle term, and the following paradigm will show the figures:

1st figure.	2d figure.	3d figure.	4th figure.
A is B	B is A	A is B	B is A
C is A	C is A	A is C	A is C
∴ C is B	∴ C is B	∴ C is B	∴ C is B

Now if we take the first four classes of propositions these may be combined two and two as premises, and hence the first figure will give sixteen MODES of syllogising according to the different manner in which we combine these four classes of propositions and so with the other figures. The following paradigms will show the manner of combining the first four classes of propositions in all the figures:

FIRST FIGURE.

First mode.	Second mode.	Third mode.	Fourth Mode.
A & B are homon, C & A " homon. ∴ C & B " similia.	A & B are hetera, C & A are hetera, ∴ C & B are hetera	A & B are similia C & A are similia, ∴ C & B are similia.	A & B are differentia, C & A are differentia, ∴ C & B " differentia, or similia.
5th. A & B are homon, C & A are hetera, ∴ C & B are hetera.	6th. A & B are homon, C & A are similia, ∴ C & B are similia.	7th. A & B are homon, C & A are differentia, ∴ C & B are differentia.	
8th. A & B are hetera, C & A are homon, ∴ C & B are hetera.	9th. A & B are hetera, C & A are similia, ∴ C & B are similia or differentia.	10th. A & B are hetera, C & A are differentia, ∴ C & B are differentia or similia.	
11th. A & B are similia. C & A are homon, ∴ C & B are similia.	12th. A & A are similia, C & A are hetera, ∴ C & B are similia or differentia.	13th. A & B are similia, C & A are differentia, ∴ C & B are differentia.	
14th. A & B are differentia. C & A are homon, ∴ C & B are differentia.	15th. A & B are differentia, C & A are hetera, ∴ C & B are differentia or similia.	16th. A & B are differentia, C & A are similia, ∴ C & A are differentia.	

MODES OF FIGURE SECOND.

1st.	2nd.	3rd.	4th.
B & A are homon, C & A " similia, ∴ C & B " similia.	B & A are hetera, C & A are hetera, ∴ C & B are hetera.	B & A are similia, C & A are similia, ∴ C & B are similia.	B & A are differentia, C & A are differentia, ∴ C & B are diff. or sim.

5th. B & A are homon, C & A are hetera, ∴ C & B are hetera.	6th. B & A are homon, C & A are similia, ∴ C & B are similia.	7th. B & A are homon, C & A are differentia, ∴ C & B are differentia.
8th. B & A are hetera, C & A are homon, ∴ C & B are hetera.	9th. B & A are hetera, C & A are similia, ∴ C & B are sim. or diff.	10th. B & A are hetera, C & A are differentia, ∴ C & B are diff. or sim.
11th. B & A are similia, C & A are homon, ∴ C & B are similia.	12th. B & A are similia, C & A are hetera, ∴ C & B are sim. or diff.	13th. B & A are similia, C & A are differentia, ∴ C & A are differentia.
14th. B & A are differentia, C & A are homon, ∴ C & B are differentia.	15th. B & A are differentia, C & A are hetera, ∴ C & B are diff. or sim.	16th. B & A are differentia, C & A are similia, ∴ C & B are differentia.

MODES OF FIGURE THIRD.

1st. A & B are homon, A & C are homon, ∴ C & B are similia.	2nd. A & B are hetera, A & C are hetera, ∴ C & B are hetera.	3rd. A & B are similia, A & C are similia, ∴ C & B are similia.	4th. A & B are differentia, A & C are differentia, ∴ C & B are diff. or sim.
5th. A & B are homon, A & C are hetera, ∴ A & B are hetera.	6th. A & B are homon, A & C are similia, ∴ C & B are similia.	7th. A & B are homon, A & C are differentia, ∴ C & B are differentia.	
8th. A & B are hetera, A & C are homon, ∴ C & B are hetera.	9th. A & B are hetera, A & C are similia, ∴ C & B are sim. or diff.	10th. A & B are hetera, A & C are differentia, ∴ C & B are diff. or sim.	
11th. A & B are similia, A & C are homon, ∴ C & B are similia.	12th. A & B are similia, A & C are hetera, ∴ C & B are sim. or diff.	13th. A & B are similia, A & C are differentia, ∴ C & B are differentia.	
14th. A & B are differentia, A & C are homon, ∴ C & B are differentia.	15th. A & B are differentia, A & C are hetera, ∴ C & B are diff. or sim.	16th. A & B are differentia, A & C are similia, ∴ C & B are differentia.	

MODES OF FIGURE FOURTH.

1st. B and A are homon, A' and C are homon, ∴ C and B are similia.	2d. B and A are hetera, A and C are hetera. ∴ C & B are hetera.	3d A and B are similia, A and C are similia. ∴ C & B are similia.	4th. B & A are differentia, A & C are differentia, ∴ C & B are diff. or sim.
5th. B and A are homon, A and C are hetera, ∴ C and B are hetera.	6th. B and A are homon, A and C are similia, ∴ C and B are similia.	7th. B and A are homon, A and C are differentia, ∴ C and B are differentia.	
8th. B and A are hetera, A and C are homon, ∴ C and B are hetera.	9th. B and A are hetera, A and C are similia, ∴ C & B are sim. or diff.	10th. B and A are hetera, A and C are differentia. ∴ C & B are diff. or sim.	
11th. B and A are similia, A and C are homon, ∴ C and B are similia.	12th. B and A are similia, A and C are hetera, ∴ C & B are sim. or diff.	13th. B and A are similia, A and C are differentia. ∴ C and B are differentia.	
14th. B and A are differentia, A and C are homon, ∴ C & B are differentia.	15th. B and A are differentia, A and C are hetera, ∴ C & B are diff. or sim.	16th. B and A are differentia, A and C are similia, ∴ C & B are differentia.	

To the foregoing paradigms, we will add another in which the existences are distinguished by their names, but without regard to figure.

1st. Snow is white—homon, The foam of the seas is white—homon, Therefore, the colors of snow and of the foam of the sea are similia.	2d. This marble is not that one—hetera, The other is not this one—hetera, Therefore, the OTHER one and THAT one are hetera.
3d. The color of John's hair is like Ma- ry's—similia, Mary's is like James'—similia, Therefore the colors of John's and James' hair are similia.	4th. An apple is not a peach—differentia. A pear is not an apple—differentia, Therefore a pear and a peach are dif- ferentia or similia.
5th. Loaf sugar is sweet—homon, This loaf is not that apple—hetera, Therefore, the taste of sweet in this sugar, and the taste of that apple are hetera.	6th. Sugar is sweet—homon, This bread tastes like sugar—similia, Therefore the taste of this bread and sweet are similia.

<p>7th. Sugar is sweet—homon, Vinegar is not sweet—differentia, Therefore the tastes of sugar and of vinegar are differentia.</p>	<p>8th. THIS biscuit is not that one—hetera, This biscuit is sweet—homon, Therefore the sweet of this biscuit and the taste of that one are hetera.</p>
<p>9th. THIS apple is not THAT one—hetera, This pear tastes like THAT apple—sim. Therefore the tastes of THIS apple and this pear are similia, or differentia.</p>	<p>10th. THIS apple is not THAT one—hetera, This pear does not taste like that apple —differentia, Therefore the tastes of THIS apple and of THIS pear are differentia or similia.</p>
<p>11th. This cake tastes like sugar—similia, Sugar is sweet—homon, Therefore the sweet in sugar and the taste of this cake are similia.</p>	<p>12th. The color of the barn is like that of the house—similia, John's barn is not the barn spoken of —hetera, Therefore the colors of John's barn and of the house are similia or diff.</p>
<p>13th. The color of the barn is like that of the house—similia, The color of the stable is not like that of the house—differentia, Therefore the colors of the barn and stable are differentia.</p>	<p>14th. Sweet is not sour—differentia, Sugar is sweet—homon, Therefore the taste of sugar and sour are differentia.</p>
<p>15th. This cake is not sweet—differentia, This bread is not the cake—hetera, Therefore, the taste of this bread and sweet are differentia or similia.</p>	<p>16th. This cake is not sweet—differentia, This bread tastes like the cake—sim. Therefore the taste of this bread and sweet are differentia.</p>

Now from the foregoing paradigms, we see that like numbered modes in each figure give like results in the conclusion and that in each figure we obtain eleven categorical and five disjunctive conclusions. From homonical premises (1) we obtain similia in the conclusion; from heterical premises (2) hetera; from similical premises (3) similia; from differentia premises (4) differentia or similia; from homo-heterical premises (15 and 8) hetera; from homo-similical premises (16 and 11) similia; from homo-differentia premises (7 and 14) differentia; from similo-heterical premises (9 and 12) similia or differentia; from similo differential premises (13 and 16) differentia; and from heterico differential premises (10 and 15) differentia or similia; in all clearer categorical and five disjunctive conclusions. And of the categorical conclusions, four are similia, three are hetera and four are differentia. Now the foregoing figures with their modes exhaust the power of syllogising with the first four kinds of propositions, in the SINGULAR SYLLOGISM.

But the fifth and sixth classes of propositions may be combined with homonical and heterical propositions, in figures and modes similar to those already exhibited. And letting A stand for the middle term, as before, the following paradigm will show the combinations in the first figure. And with the fifth and sixth classes of propositions we may use the sign = equal to, between commensura, and $>$ or $<$ the sign in commensura, just as in mathematics.

1st. A & B are homon, C & A' are homon, $\therefore C=B$.	2d. A & B are hetera, C & A are hetera, $\therefore C \& B$ are hetera.	3d. A=B, C=A, $\therefore C=B$.
4th. $A > B$ $A < B$ $A < B$ or $A > B$ $C > A$ $C < A$ $C > A$ or $C < A$ $\therefore C > B$ $\therefore C < B$ $\therefore C=B$ or $C < B$ or $C > B$		5th. A and B are homon, C and A are hetera $\therefore C$ and B are hetera.
6th. A and B are homon C=A $\therefore C=B$.		7th A and B, are homon, C < A, or C > A $\therefore C < B$ $\therefore C > B$.
8th. A and B are hetera, C and A are homon, $\therefore C$ and B are hetera,		9th. A and B are hetera, C=A $\therefore C=B$ or $C < B$ or $C > B$.
10th. A and B are hetera, C > A or C < A, $\therefore C=B$, or $C > B$, or $C < B$,		11th. A=B, C and A are homon, $\therefore C=B$.
12th. A=B, C and A are hetera, $\therefore C=B$, or $C < B$, or $C > B$.		13th. A=B, C < A, or C > A, $\therefore C < B$, $\therefore C > B$.
14th. A > B, or A < B, C and A are homon, $\therefore C > B$ $\therefore C < B$,	15th. A > B or A < B, C and A are hetera, $\therefore C=B$, or $C < B$, or $C > B$.	16th. A > B or A < B, C=A C=A, $\therefore C > B$ C < B.

We do not deem it necessary to give paradigms of the modes of the remaining three figures in which homonical, heterical, commensural and in-

commensural propositions are combined. Now the four figures with their modes, in which the first four classes of propositions are combined and the four figures with their modes, in which homonical heterical, commensural and incommensural propositions are combined, exhaust the whole power of syllogising in singular syllogisms, i. e., in comparing two existences by the means of an homonical existence or of two similical or commensural existences. We have not put the words—ALL, EVERY, NO &c., before any of the terms, because these words, as we have heretofore shown, do not change the character of the affirmation, but belonging to the terms they are used to distinguish and characterise the existence, which we are comparing, and they may be thrown out of every proposition, in which they occur, excepting numerically complex propositions, by changing the wording of the proposition and without affecting the result; as ALL men are mortal, is equivalent to man is mortal, i. e., one of the gregaria sine qua non of man and mortality are homon. The propositions, ALL the Apostles were Jews; ALL the boys in the room are barefooted, &c., are numerically complex propositions, and they are not used in the singular syllogism. The words—SOME, MOST, a FEW, &c., also distinguish merely the numerical relations inter similia upon a certain generalization. And by the custom of our language, every proposition, in which they occur, may be stated in other words, which shall not express, but imply their substance; as SOME apples are sour, into ALL apples are not sweet; i. e., sweet and sour are not gregaria sine qua non of apples. And hence, SOME, MOST, a FEW, &c., show, in propositions, an indefinite numerical relation among apples, for instance, which as apples are similia, but which, outside and over and above the gregaria, sine quibus non, possess other gregaria, which, when considered, enables us to distinguish and further differentiate.

CHAPTER XVIII.

EXPLANATION OF THE SYLLOGISM.

If a man were in a wood among fallen timber and found two logs, which he was unable to lift, and whose comparative lengths he desired to know, without the use of the syllogistic process he would not be able to accomplish his object. If however, he should cut a rod, which we will call A, he could go with it to the first log, which we will mark 1st, and find that 1st and A are commensura, and then with his rod he could go to the second log, which we will mark 2d, and find that 2d and A are commensura and then he would have the premises of a syllogism: 1st and A are commensura, 2nd and A are commensura, therefore 1st and 2d are commensura; or $1st = A$, $2d > A$, if it be so and therefore $2d > 1st$. And without the power to syllogise the carpenter could make no use of his foot-rule, the shoemaker no use of his last, the

farmer no use of his half-bushel; no one could put into a pile one cord of wood; and no one could tell without first having knocked his hat off, whether the door in his house was high enough to let him enter without bending his body. The process of syllogising is used by every person in the daily vocations of life, and it always has been so used from the creation of man.

But notwithstanding the almost constant use of the syllogism by all men, the process itself has been misunderstood both by the friends and the enemies of logic. The opposers of logic have represented that if the syllogism be a true process of reasoning used by us in matters about which we reason, men could not have reasoned at all before the time of Aristotle, who is regarded as the true expounder of logic; which is argument is analogous to the following; If the wheels of a wagon turn upon the principles of the lever before these principles were understood men could not have driven wagons. The contempt, however, which the opposers have heaped upon logic, and of which its friends complain, is not owing to the want of a syllogistic process in the mind, but to the circumstance that the friends of logic have been neither able to explain this process, nor to refute the objections of its adversaries.

For the explanation of the syllogism, most of the writers upon logic have relied upon the Aristotelean dictum *de omne et nullo*—what ever can be predicted of a class can be predicted of any individual of that class—and hence they say that the middle term must always be distributed in one of the premises by being the subject of a universal affirmative or the predicate of a negative proposition, which in our opinion amounts to nothing so far as the syllogistic process itself is concerned. For a class is nothing else than several individuals *inter se similia*, or but one individual differentiated from all other things; and hence the dictum asserts merely that whatever can be predicated of each one of *similia* can be predicated of any one of *similia*; and although this is true, it is but a part of the whole truth. If we have before us several marbles, the colors of which are *inter se similia*, we may with equal truth, turn the dictum the other way, and say that whatever can be predicated of the color of any one of the class, can be predicated of the color of each one of the class, for the reason that the colors are *inter se similia*. And for the same reason and for none other, to-wit, that the individuals are *similia* in the respect in which every one or any one is spoken of, or joined with a certain predicate in a proposition, does the dictum mean anything: that there actually are in nature *similia*, *differentia*, *commensura* and *incommensura*, is the foundation of the dictum, and yet a syllogism may be constructed of homonical or heterical premises. And from the notion that in every syllogism the middle term must be distributed in one of the premises, i. e., stand for a whole class of individuals *eo nomine et in numero*; while in truth it is never does so stand, but always represents an homonical

individual, or an individual of similia, or of commensura, the friends of logic have been overpowered by their own logic. And hence the friends of logic have conceded to its adversaries, that in every legitimate syllogism, the conclusion contains nothing which is not employed and virtually asserted in the premises. For say they we reason from generals to particulars, and what is true in general is true in particular—dictum de omni et nullo. And although J. Stuart Mill was able to see that Aristotle's dictum was only adapted "to explain in a circuitous and paraphrastic manner the meaning of the word class." Yet he too along with the rest was overpowered by the dictum. And hence he says "It must be granted that in every syllogism considered as an argument to prove the conclusion, there is a *petitio principii*. When we say all men are mortal, Socrates is a man, therefore Socrates is mortal, it is unanswerably urged by the adversaries of the syllogistic theory, that the proposition, Socrates is mortal, is presupposed in the more general assumption, all men are mortal; that we could not be assured of the mortality of all men, unless we were previously certain of the mortality of every individual man; that if it be still doubtful whether Socratee, or any other individual you choose to name, be mortal or not, the same degree of uncertainty must hang over the assertion, All men are mortal; that the general principle, instead of being given as evidence of the particular case, can not itself be taken for true without exception, until every shadow of doubt which could affect any case comprised with it is dispelled by evidence aliunde and then what remains for the syllogism to prove? That in short, no reasoning from generals to particulars can, as such, prove any thing; since from a general principle you can not infer any particulars but those which the principle itself assumes as foreknown. This doctrine is irrefragable."

Now this "irrefragable doctrine" is owing to a misconception of the nature of propositions and of their combinations in the syllogism. In the first place it is not true, although it has generally been conceded to be so, that there is nothing contained in the conclusion, which is not implied in the premises. In the syllogism, A and B are similia, C and B are similia, therefore C and A are similia, we have indeed the existences A and C in the premises, their relation, however, to each other, is neither expressed nor implied in either of the premises, but it is evolved from the COMBINATION of the premises. And if it be meant that by the COMBINATION of the premises the conclusion is implicated, this indeed is true, but this certainly can not be urged as an objection, for it is of itself an approval of such combination for the purpose of gaining a result, which we can not obtain without such combination. In order to understand this matter clearly, it is necessary that we enter into an elaborate explanation of the syllogism. We have shown heretofore that when the existences really compared in any proposition are clearly set out by the wording of such proposition, the terms of the proposi-

tion may be transposed; as all men are mortal, i. e., one of the gregaria sine qua non of man and mortality are homon, and by transposition, mortality and one of the gregaria sine qua non of man are homon. And hence when a proposition is so worded that the terms may be transposed (and every proposition can and ought to be so worded when it is considered in a scientific view) it may be combined with another proposition worded in like manner, in any one of the four figures; and therefore, an explanation of the syllogism in any one of the sixteen modes of any figure, will be an explanation of the like numbered modes in all the figures.

We will commence our examination, therefore, with mode 1st in the paradigms in which the first four kinds of propositions were used. Take the syllogism, All snow is white or snow is white, The foam of the sea is white, therefore the colors of snow and of the foam of the sea are similia, i. e., snow and the foam of the sea are similia in one facial gregarium—color—which facial gregarium of snow and that of the foam of the sea, have each of them been differentiated from the other four nominal truths into color; but inter se they could not be differentiated, and therefore they are similia. But we have heretofore shown that hetera lie at the very foundation of our knowledge. Suppose then that we look at the color of paper, and without any reference to discrimination say—this is—and having turned our eyes away from it, look at the same paper again and say—this is; now is this the thing which, we have said, is, when considered as grounded in the ego, the same thing in both cases? certainly not; and why not? Simply for the reason that their times can be heterated, and the power of our minds to heterate, gives us the knowledge that THEN and NOW are hetera and that an existence grounded in the ego five minutes ago is not subjectively the same existence grounded in the ego now. But if two existences can be heterated only, the two must be to us inter se similia; and therefore when we have said, THIS IS, and THAT IS, if we can discriminate no farther we must say, THIS and THAT are similia, and merge the two homonical propositions into one similical proposition. Returning therefore to the premises, Snow is white, The foam of the sea is white, the heterical WHITES are similia we can discriminate them no farther than into hetera, and hence the conclusion must follow that the color of snow and that of the foam of the sea are similia. But when we say, Snow is white, The foam of the sea is white, therefore the colors of snow and of the foam of the sea are similia, we must recollect that the heterical WHITES, which are subjectively similia, have, each of them, an objective where, and therefore they are also objectively similia, while if we should project them into an homonical where, they would be objectively homon. The above premises, therefore, contain four subjective existences, two of which the heterated whites, are subjectively and objectively similia; objectively however, there are but two existences in the

premises to wit, the color of snow and the color of the foam of the sea; and objectively the syllogism in mode 1st, in the conclusion locates these objective existences, as similia in their respective WHEREs.

Mode 2d, if we consider the four heteretical existences of the premises merely subjectively, they would not bring us into a conclusion; but two of the subjective existences must be considered as occupying an homonical where in an homonical time; they must be objectively homon. When we say 1st and 2d are hetera, 3d and 2d are hetera, therefore 1st and 3d are hetera, the two subjective 2ds must be referred to an homonical where at an homonical where at an homonical time; but 1st and 3d, cannot be homon for they are not compared with each other in either of the premises, but they are brought together by means of 2d, and if 2d and both 1st and 3d, be hetera, as stated in the premises, 1st and 3d must also be hetera. It may, however, be said that in this mode the conclusion does not follow from the combination of premises; for, if we put before us three objective existences, marked 1st 2d 3d, we can say first is not third, without comparing each of these with second. This is true; but it is the distinguishing terms, 1st and 3d, which enable us to jump the middle existence. Suppose we apply our nose to a rose and say This (1st) smell is not that scent, we then apply our nose again and say, This (2d) smell is not the 1st smell, therefore 2d smell and THAT scent are hetera. In this case the 1st smell, which is the middle existence appears twice subjectively, but we refer these two subjective existences to an homonical where and time, and therefore they are homon, and without this middle existence we could not gain the conclusion, that second smell, and THAT 1st SCENT, the homonical scent mentioned in the first premise, are hetera.

In mode 3d, each of the premises is a conclusion drawn from a former syllogism: as A is white, B is white, therefore the colors of A and B are similia, (mode 1st); A is white, C is white, therefore the colors of A and C are similia (mode 1st); and from these conclusions we form the premises, A and B are similia, C and A are similia, and hence C and B are similia—conclusion. Mode 3d needs no further explanation.

Mode 4th is somewhat more difficult. When we say, sweet is not sour, bitter is not sweet, we are apt to look back at the words sour and bitter, and as these words distinguish differentia, we see from the terms that SOUR and BITTER are differentia, and hence we are apt to infer merely differentia from the premises. When we say, A peach and a pear are differentia A potato and a pear are differentia, we will naturally say, A potato and a peach are differentia, which indeed is true, but it is not THEREFORE true, it does not follow from the premises. No categorical conclusion can be legitimately drawn from these premises, the conclusion which really does follow, is that a potato and a peach are either differentia or similia. This will easily be seen if we treat a peach and a potato merely as hetera and call the peach FIRST, and the

potato SECOND: then dismissing from our mind those differential names, we say, 1st and a pear are differentia, 2d, and a pear are differentia, and as we do now see from the terms 1st and 2d whether they be differentia or not, the conclusion follows legitimately in our minds from the premises, and we conclude that 1st and 2d are differentia or similia.

The fifth and eighth modes, which are in substance alike, are easy. The color of THIS marble is white, the color of THIS marble and the color of THAT one are hetera, therefore the color of that one, let it be what it may, and the white in the first marble, are hetera. Snow is white, snow and paper are hetera, therefore the color of snow and the color of paper are hetera, i. e., snow has a color and paper has a color and the two colors are hetera.

The sixth and eleventh modes, which are similar in substance, contain greater difficulties. When we say this apple is sweet, that pear tastes like this apple, it is quite clear that the conclusion, therefore, that pear is sweet, follows from the premises, though this conclusion is an homonical proposition. The taste of this apple and sweet are homon, the taste of this apple and that of that pear are similia, therefore the SWEET in the apple and the taste in the pear are similia; but similia have a common name, and therefore the taste in the pear when named, is called sweet, and we say in the conclusion, that the pear is sweet, i. e., that the taste of the pear and sweet are homon. Now if we examine the above syllogism closely, we will see, that in the premises there are subjectively four heterical existences, to-wit; 1st, The taste of this apple; 2nd, Sweet; 3d, The taste of this apple; and 4th, The taste of that pear; three of which subjective existences are objectively homon. THE TASTE of that PEAR only, is located in an heterical where with reference to the where occupied by "The taste of this apple, sweet, and the taste of this apple;" the other three heterical existences of the premises subjectively; but objectly these three are homon. But the SWEET mentioned in the conclusion is not objectively homonical with the sweet in the first premise, they are objectively similia, and because they are similia they have a common name and we say This pear is sweet, i. e., one of the gregaria of this pear and sweet are homon. Therefore in modes 6th and 11th there are but two objective existences in the premises, which are inter se similia, and in the conclusion, one of these similia appears located in one of the objective WHEREs mentioned in one of the premises, as the other one of the similia was located in the other WHERE in the other premise. In mode first we saw that of the four subjective existences in the premises, the two in the first premise were homon, and the two in the second premise were homon; in modes 6th and 11th, the two subjective existences in one preinise and one of the subjective existences in the other premise, are objectively homon. And we must see hat if we take the conclusion, The sweet in this apple and the taste in that pear are similia; and dress it in common language, viz: That pear is sweet,

and then combine this conclusion with the homonical proposition of the above premises, and we will be in mode first, and will gain the other premise of the above syllogism as the conclusion: The taste of this apple is sweet the taste of that pear is sweet, therefore the tastes of the pear and apple are similia. And to make the matter still clearer, we may suppose three persons, whom we will call A, B and C, to be sitting in a room with two apples in their hands. A tastes both of the apples and says secretly to himself, "this apple is sweet and that apple is sweet," and then drawing the conclusion in mode 1st, he says aloud, "this apple tastes like that one;" B then tastes one of the apples and says, "this apple is sweet;" well then says C from what A and B say, "the other apple is sweet also."

But hitherto we have not used what are called universal propositions for either of our premises, and when general propositions are used in mode 1st, it is then, that a *petitio principii* is supposed to occur. We did not discuss this matter when treating of mode 1st, for the reason, that we desired to get the reader further along in the knowledge of some of the other modes, so that he might be better prepared for such discussion. When we say, all men are mortal, Socrates is a man, and, therefore Socrates is mortal, it is said that the conclusion, Socrates is mortal is implied in the first premise, All men are mortal. The difficulty in this syllogism is, indeed somewhat below the surface, but if we set clearly before us the existences, which are really compared in the premises, the solution will be more easily obtained. All men are mortal, or its equivalent, Man is mortal, shows that one of the *capacial gregaria sine qua non* of man and mortality are homon; Socrates is a (one) man, shows that the existence called Socrates and one of the existences called man are homon; and therefore Socrates, who is homonical with one man, and other men are similia, in mode 1st. The SIMILE, mortality, exists in every object, which may be called man, but Socrates, i. e., the object designated by that name, may be called a man, and therefore this simile exists in Socrates; for MAN is the common name of similia. In the foregoing syllogism let us write the premises and conclusion thus: Socrates and A man are homon, One of the *gregaria sine qua non* of man and mortality are homon, Therefore the *gregaria sine qua non* of man and the *gregaria* of Socrates are similia, and One of these *gregaria* of Socrates then must be mortality, Socrates must be mortal.

Suppose we look back to what we have called nominal truths, where we saw that when an object of vision arose into consciousness we called it COLOR, to distinguish it from conscious truths of the other senses; and suppose that the first object of vision should have been the color, which we now call red; red then would have been called color, to distinguish it from conscious truths of the other senses. Then suppose green to have arisen into consciousness, green too would have been called color, to distinguish it from

objects of the other senses, and then red and green, as color, as distinguished from objects of the other senses, are inter se similia, and therefore each of them is a color. Now, if we collect into an homonical proposition the very thing, which enables us to differentiate objects from other things into colors, to-wit, visibility, we will say, All colors are visible, or its equivalent, Color is visible, i. e., Color and visibility are objectively homon, and if we then add That red is a color, i. e., Red and one color are homon, it will follow that the object called RED and visibility are similia i. e., red as an object distinguished from conscious truths of the other senses is distinguished in the same manner as other colors, to-wit, by being visible. And we must perceive that the first premise gives visibility as the ground of differentiation from the conscious truths of the other senses, the whole of which ground lies partly in the visual faculties and partly in external objects, that is, in the relation of these, and it gives also COLOR as the name to distinguish that part of the ground lying in external objects; and hence color and visibility are objectively homon. The second premise takes one of the subjective similia so differentiated, and pronounces this SIMILE and RED, a color further distinguished among colors to be homon; and hence this simile and any other simile are similia (non simile est idem) and red as a color and visibility, when located in the same where, are homon, for similia have a common name and when their wheres are homonical, they are objectively homon.

Again, suppose we take several sticks, each one of which we dot with differently colored dots in such manner that by looking at the sticks when thus dotted, we cannot by the dots discriminate one stick from another, and suppose that each dot on any stick can be discriminated from any one of the other dots on the same stick, and to distinguish the dots inter se, we call one A, another B, C, D, &c. Now letting the dots in the aggregate be the very things, which distinguish the sticks before us from other things, we will call these dots, in the aggregate, in fasceculo, A. But supposing that by the lengths of the sticks we are able to distinguish the sticks inter se, we will call a particular stick B, another C and another D. Now we can say that one of the dots of every A is A, i. e., one of the dots of any A and A are homon. But B, this particular stick, which I now hold in my hand and mention by the name B, is a (one) thing, whose aggregate dots are called A, i. e., B and one of the A's are homon, therefore any one of the A's excepting the A which I hold in my hand and mention by the name B, and the A which I hold in my hand and which is the same thing as B, are similia; and hence the homonical A which we find in any A excepting the A, which is also B has a simile, a dot like itself, in the A in my hand which I may call also B, B is A.

It must be confessed that the exposition of this matter is some what difficult; and heretofore all logicians have failed to understand the true state

of the case, but by thinking over the matter for several times, we hope the reader will be able to see through it. Perhaps it will appear more clear to some minds, if we dismiss differential terms for the aggregate existence, and distinguish them merely as hetero; then one of the gregaria sine qua non of 1st object and mortality are homon; let this be our first premise. And then it must appear that if we say a second object and the 1st are similia, it will follow in mode 6th, that the simile mortality located in the first objects has a SIMILE located in the second one and this SIMILE is mortality. But if after the first premise, we say that the 2d object is one of the first kind of objects, this proposition, though homonical, is quasi similitical, and the conclusion from the homonical premises that the gregarium mortality located in the 1st object has a simile in the second one is quite evident, and this simile located in the second object must be called by the common name, mortality, and hence one of the gregaria of 2d object and mortality are homon. All men are mortal, i. e., one of the respects in which men are similia and mortality are homon; Socrates is a man, i. e., The object called Socrates and one of the similia named man are homon; therefore the respect, to-wit, mortality, in which men are similia and which is a gregarium in other men, and this respect in the object called Socrates, since he is a man—Socrates is mortal.

The reason that syllogism, like the above are so difficult to understand, is that we lose sight of the WHEREs in which the RESPECTS, the gregaria, which render objects similia, exist. When we say, Snow is white, the snow in which this gregarium WHITE, exists, or did exist, has or had an objective WHERE, but this where is indefinite and undistinguished in our minds from other wheres. But when we announce to a friend in the street that Snow is white and then add that an object in our house, which object the friend has never seen nor heard of before, is snow, he will immediately conclude that the colors of the object in our house and of the snow located in an indefinite WHERE are similia, and therefore he would say that the object in our house is white.

Now we do not concede that this argument is a *petitio principii*, that when we say all snow is white, we imply that the object in the house is white; before this conclusion can be reached, without seeing the object itself, we must first learn that the object in the house and snow in the respect of color, are similia, and this we do when we are informed that the object in the house and one of the similia named snow are homon. So when we say all men are mortal, we do not imply anything respecting the object named Socrates, for Socrates may be the name of a statue or of a fictitious god like Jupiter. In the syllogism, all men are mortal, Socrates is a man, and therefore Socrates is mortal, however, both premises as they are usually understood, and the conclusion, are false. Iron already fused is not fusible unless it be first congealed again; neither are dead men mortal, *requiem eternam Domine da eis*.

In the syllogism, All men are mortal; All kings are men, therefore all

kings are mortal; mortality is one of the *gregaria sine qua non* of man, and man is a *sine qua non* of a king, and therefore mortality is a *sine qua non* of kings. It may be said, indeed, that when we say All iron is fusible; so soon as we say of any object that it is iron, we have already in the first premise asserted that it is fusible, and it is true that by the combination of the premises we reach the conclusion: and this is the case in every syllogism, whether either of the premises be a universal proposition or not. When we speak of particular objects and say A and B are similia, so soon as we say A and C are similia, we bring B and C to be similia, yet there is no *petitio principii* about it.

Now when we say Man is mortal, we mean that one of the *gregaria sine qua non* of man and mortality are *homon*: but when we say Man is A mortal, we mean that each man and one of the similia, each one of which is named a mortal or mortal being, are *homon*: and this proposition brings man among the similia called mortals, in each one of which there exists the *SIMILE*—mortality. We have perhaps gone far enough with the explanation of this matter.

Modes 7th and 14th are very easily understood: Sugar is sweet—*homon*; No vinegar is sweet, or Vinegar is not sweet—*differentia*; Therefore the tastes of sugar and vinegar are *differentia*. The 9th and 12th modes are easy: and after having gone through the previous explanations, we do not deem it necessary to consider the remaining modes, as the principle of each of them has already been exhibited in some of the foregoing explanations. It may, however, be well enough, in order that the reader may have a clear understanding of our system, to take a view of those rules which writers generally have laid down for the regulation of the syllogism.

And in order that the reader may better understand the whole matter, it must be observed that logicians have divided propositions into universal affirmative, as All men are mortal, which class of propositions they distinguish by the symbol A; universal negative marked E, as No gold is green; particular affirmative marked I, as Some islands are fertile; and particular negative marked O, as Some men are not black. And with these four classes of propositions they commence to syllogize and to construct rules for obtaining true conclusions.

And the first rule which they give, is that Every legitimate syllogism must have three and only three terms—the middle and the two terms of the conclusion. Although this rule, if we look merely at terms, be true, yet we consider logic to be concerned about more than terms, and therefore, we state instead of this rule that In every legitimate syllogism, there must be four and only four subjectively heterical existences in the premises, two of which—one in each premise—must be objectively heteral, and the two of which with which the other two are each compared, must be objectively *homon*, or *similia* or *commensura inter se*.

The second rule which they give, is that Every legitimate syllogism must have three and only three propositions: in this we are agreed.

The third rule which they give, is that The middle term must not be ambiguous. This danger is sufficiently guarded against by our first rule respecting every legitimate syllogism.

The fourth rule which they give, is that The middle term must be distributed once at least in the premises by being the subject of an universal affirmative or the predicate of an universal negative proposition. For, say they, if we say white is a color, black is a color, in which propositions the middle term—A COLOR—is not distributed, we will conclude falsely that black is white. But after what we have said heretofore, we think, it will readily be perceived that both of the above premises are homonical propositions and that the predicates of each—a color—are objectively two and not one and the same existence, they are not homon, and that these two existences have been differentiated from existences of the other senses, into colors, in which class of existences as distinguished from other things, as nominal truths, they are similia, the name color will distinguish either of them from existences of the other senses. When therefore we say, white is a color, black is a color, it does not follow that white is black, but that white and black as distinguished, not inter se, but from other things are similia. White is A COLOR, black is A COLOR, therefore white and black, as nominal truths, are similia. But it does not follow that inter se, white and black are similia, unless it appear that the predicate, A COLOR in the first premise, and the predicate, A COLOR in the second premise are inter se homon, or similia; the middle term therefore is faulty, not because it is not distributed, but because two existences are used which do not appear to be inter se similia. The fourth rule, therefore, laid down by writers, as a guide to keep us upon the true process of the mind in syllogising correctly, we conceive to be, not only of no value, but erroneous.

The fifth rule given, is that No term must be distributed in the conclusion, which was not distributed in one of the premises. "All quadrupeds are animals, a bird is not a quadruped, and therefore a bird is not an animal." This conclusion is evidently erroneous; and it is quite clear that those, who were engaged in the construction of this rule, saw, independently of the syllogistic process in the premises, the error in the conclusion, which from the appearance of the words in the premises might be supposed to follow legitimately. The proposition, "All quadrupeds are animals." means simply that each quadruped and one animal are homon, and when we add that a bird is not a quadruped, i. e., that each bird and any quadruped are differentia, it does not follow that each bird and any animal are differentia, what follows legitimately, is that each bird and the animals homonical or similical with the animals included in the predicate of the homonical pro-

position "all quadrupeds are animals" are differentia. For bird and animal are brought into the comparison in the conclusion by means of an homonical existence or similital existences, with which they were each of them compared in the premises. We stated in our first rule that each existence, which appears in the conclusion, must be compared in the premises with the same middle existence or with two existences inter se similia or commensura. And in the above premises quadruped is compared with one animal, and quadrupeds being inter se similia, bird is then compared with one of these similia, and the conclusion must be that the animal compared in the homonical proposition and found to be one of the quadrupeds and every bird must be differentia, but nothing can be inferred respecting any other animal, except it be a simile, than the animal spoken of in the first premise, which was homonical with quadruped. Red is a color, Green is not red, are premises just like the former, and from them it follows that the one color homonical with RED and green are differentia. The fifth rule therefore is of no value in our system, it is erroneous and fallacious as a grade in the syllogistic process.

The sixth rule given, is that From negative premises you can infer nothing. This rule in our system has no meaning, for, we do not admit that there is any such thing as an independent negative proposition. But calling such propositions, which have NO, NONE and NOT in their negative, the rule itself is not true, it is only true that we can not infer a categorical conclusion. From the premises "A fish is not a quadruped, A bird is not a quadruped," it legitimately follows that a fish and a bird are differentia or similia (mode 4th).

The seventh rule given is that if one premise be negative the conclusion must be negative. This rule in our system means nothing.

Now in stating every homonical proposition, such as All men are mortal, we must be careful to see whither the predicate be one of the gregaria of the subject or not; for if it be not, and it be represented by an adjective name in order to make the proposition clear, some noun must be placed after it, or understood for adjective names which are not the representatives of gregaria, are the names of existences standing as a class by themselves. When we say "All gold is precious," we mean that all gold and one of the things esteemed of value among men, are homon; the proposition therefore should be stated thus; All gold is a precious thing, and then we can add that All gold is a mineral, and it will follow that the mineral homonical with gold is a precious thing. Mr. Hamilton gives as the second rule, that "The subsumption must be affirmative," and he illustrates this rule by the following example; "All colors are physical phenomena, no sound is a physical phenomena;" "Here" says he, "the negative conclusion is false, but the affirmative, which would be true—all sounds are physical phenomena—can not be inferred from the premises, and therefore no inference is competent at all."

(page 289) After what we have said heretofore, I think, it will be very easy to see through Hamilton's mistake. When we say that "All colors are physical phenomena," we mean that each color is a (one) physical phenomenon, and when we add, No sound is a color, we mean that any sound and any color are differentia, and therefore we can infer, not that no sound is a physical phenomena, but that all physical phenomena homonical with colors and sounds are differentia. We have gone far enough perhaps, in this direction to make ourselves understood by the reader.

Before leaving this chapter, however, it seems necessary, that we should make some remarks tending in another direction. It is the unanimous doctrine of logicians hitherto, that one of the premises at least must be what they call a universal proposition, otherwise no legitimate conclusion can be drawn. And hence, if we should take a stick and apply it to a table and find the lengths of the stick and table so be commensura,* and then apply the stick to another table and find the stick to be longer than it, and we should then make the following statement; 1st table=stick, 2d table<stick, therefore 1st table>2d table, this would not according to the received doctrine be a legitimate syllogism. But if this be not a legitimate syllogism, what is it? General propositions are necessary at all to enable us to syllogise, excepting when we wish to syllogise with gregaria or a gregarium sine qua non of objects. When we say all A is b, i. e., one of the gregaria sine qua non of A and b are homon, no B is b, i. e., the gregaria sine qua non of B and b are differentia, it follows that A and B are differentia. In such cases as these, general propositions are necessary; but such cases form but a part of the instances, in which the syllogistic process is used. And from the consideration no doubt, that general propositions are always necessary in order to be able to syllogise, J. Stuart Mill, concluded that the syllogistic process was not really inferential reasoning. He says "In the above observations it has, I think, been clearly shown, that, although there is always a process of reasoning or inference, where a syllogism is used, the syllogism is not a correct analysis of that process of reasoning or inference; which is, on the contrary, (where not a mere inference from testimony) an inference from particulars to particulars: authorized by a previous inference from particulars to generals and substantially the same with it; of the nature, therefore, of induction." Now when we tell a friend that the heighth of a stove in this room is commensural with the heighth of a stove in the other room, which latter stove the friend has never seen, and that the heighth of this stove is three feet, and then ask him from these data to tell us the heighth of the stove in the other room, if he does not syllogise and on the syllogistic process make an inference, I should like to know in what other manner, by what kind of induction, he would be able to solve the problem.

CHAPTER, XIX.

EXPLANATION OF SYLLOGISM CONTINUED.

Having explained the syllogism, in which the first four classes of propositions are combined, we come now to give some further consideration to the syllogism combining the first and second and fifth and sixth classes of propositions. And of the manner, in which the first and second classes of propositions are combined in the syllogism, we have already said sufficient; it is to the manner of combining commensural and incommensural propositions, therefore, that we will more especially direct the attention of the reader. In our explanation of propositions heretofore, we observed that, similical and differential propositions spring from homonical propositions; we showed this to be the case also with commensural and incommensural propositions. Homon is at the bottom of all propositions; hetera are at the bottom of all knowledge; and the power of the mind to HETERATE depends upon time and space. We must also perceive that, homonical propositions, which are collected into heterical, similical, differential, commensural or incommensural ones, must in every instance have a local reference in the subject or predicate; for, in every proposition there is a comparison between two existences, and if these two existences be considered merely heterically, they can not subjectively be homon; to be homon the subjective hetera must be located in an homonical where at an homonical time. We have already seen, how we come to have the knowledge of existence; and after this has been obtained, we may say indeed, that THIS grounded in the ego and one existence grounded in the ego are homon; but when the ONE EXISTENCE is grounded in the ego, it is located there in the same where, with THIS, and at an homonical time; and the ONE EXISTENCE and the THIS referred to must also, irrespective of time and space, be subjectively similia, otherwise the bringing them into an homonical where at an homonical time will not make them homon. An object may be heard by the ear and another seen by the eye; irrespective of time and space they are differentia, and although they may subjectively be located in the same where at an homonical time they do not become homon. And where we say, This is an existence, and then again, That is an existence, the first existence and the second one are hetera, and if they can not be discriminated further they are similia. Existences, however, is a name, which does not distinguish existences inter se. But if we say, This is white, and then again, THAT is white, as white is a name, which distinguishes existences inter se, if the first thing and the second thing be not similia, in the respect of color the word, WHITE has been misapplied to one or both of them.

Now in commensural and in incommensural propositions, the things compared are always similia, yet commensural and incommensural propositions are not derived from similia but from homon. If we take a certain stick and say, the length of this stick is ONE (the unit) i. e., the length of this stick and ONE are homon, and we then go to an other stick and say, the length of this stick and ONE are homon, if the lengths of the two sticks be not commensura, ONE has no definite meaning; and we can give a definite meaning to ONE only by taking some homonical thing as the unit of measurement. If then we make the length of a particular stick the homonical thing by which to define ONE, and apply this length to another stick, and we can not discriminate the lengths of the two, we may say, the length of the first stick, the homonical thing which we have made the unit of measurement, and ONE are homon, and as the length of the second stick when compared with the first cannot be discriminated from it, we must from a mental necessity call it ONE also. The length of the first stick and ONE are homon, the length of first stick and that of the second are commensura, therefore the length of second stick and ONE are commensura; but commensura must of necessity have a common name, and hence the length of second stick must be called ONE, and length of second stick when not compared with another, and ONE are homon. And if we combine this proposition, with the homonical one which gave the unit of measurement, we will have length of first stick and ONE are homon, length of second stick and ONE are homon, therefore lengths of first and second sticks are commensura, since one and one (not twice one) are commensura, $1=1$; and hence the length of any stick which may be called one, will be commensural with the first stick. If however, the length of first object $< 2d$ and $2d < 3d$, then $1st < 3d$, and we have three heterical objects, which are inter se incommensura, and we may continue by sorites, $3d < 4th$, therefore $1st < 4th$, but $4th < 5th$, therefore $1st < 5th$, but $5th < 6th$, therefore $1st < 6th$, and therefore 1st or any of one of the objects after 1st, is less than 6th, and so on. Here then we have six objects inter se incommensura, and as they are similia in kind, each of them in a like manner, has been differentiated from other things, and they have a common name distinguishing them from other things in kind; but this name does not distinguish them inter se. And if we name them 1st, 2d, 3d, &c., these distinguishing terms merely distinguish them heterically inter se, but they do not show the incommensural relations existing among them, and therefore by the use of such terms, we can not show any results further than heterical, which we may have obtained by comparing those objects inter se. There is therefore, only one possible way for us to form a language by whose terms, we may be able to show the results of the minds comparisons among commensural and incommensural objects. After that we have gained the knowledge of the homonical thing, which we establish, as the unit of measurement

in an homonical proposition, we may apply this homonical unit to a second object, and if the homonical thing be measured just twice upon the second object, we may arbitrarily name twice one, TWO, and then twice one and two will always be in our minds commensura, and TWO will show the result of the comparison between any object named TWO, and the homonical thing called ONE. And by naming thrice one, THREE, four times one, four, and so on, we will have the cardinal numbers applied to similia. One, then, will be a common name for all objects, which are inter se similia and inter se commensura; and so also will 2, 3, 4, &c. But 1, 2, 3, 4, &c., distinguish incommensura inter se, and show by the relations of the homon inter hetera, the incommensural relations existing among similia. And these arbitrary signs of commensural and incommensural relations may be applied to any similia in nature, by taking an homonical simile as the unit of measurement; they may be applied to lengths, to heats, to colds, to weights, to volumes &c. It is the peculiar prerogative of mathematics to develop and carry out these principles.

But we must see that the unit of measurement, in all cases, is the predicate of an homonical proposition, and then commence commensural and incommensural propositions. And the syllogism with commensural and incommensural propositions, is used in every branch of mathematics from the beginning to the end. And as the demonstrations, in mathematics depend upon definitions, it is necessary to consider the manner in which we syllogise upon those definitions. We have, heretofore said, that all definitions, which state directly what a thing is, are contained in homonical propositions; this is the case in mathematics, and as geometry affords us sufficient illustration of our subject, we will confine our remarks to it. Geometry, it needs not to be shown here, treats of relations in space, and hence A POINT is a position, a where in space, i. e., a mathematical point and a where in space are homon. A line is the cause of consecutive points in space. A straight line is the course of consecutive points in a uniform direction in space, i. e. a straight line and a course of consecutive points in a uniform direction in space are homon. And again, the portion of space included between two lines touching each other at a given point, and an angle are homon. Again, the portion of space, which being included by two straight lines touching at a given point, which point being taken as a center and a circle described, is a quadrant of the circle, and a right angle are homon, and so on. All the foregoing definitions, and all of the direct definitions upon which in geometry demonstrations are constructed, are contained in homonical propositions. But when we say, an acute angle is an angle less than a right angle, we do not directly define an acute angle, and therefore the proposition is an incommensural one, and so also when we say, an obtuse angle is greater than a right angle, And it must be observed that LINE is a common name for simi-

lia and that STRAIGHT LINE is also a common name for similia, LINE being a genus of which STRAIGHT LINE is a species; and so also with angles &c. But after the definitions in geometry, then follow what are called axioms. These axioms are contained in commensural and in incommensural propositions, the bottom of which, as we have seen, is homon. But the commensural and incommensural propositions which contain axioms are founded more immediately upon the syllogism. The axiom, that Things inter se similia, which are equal to the same thing are equal to each other; is obviously the condensation of a syllogism into a commensural proposition. Let the length of a certain stick be the homonical unit of measurement and call this length ONE; ONE then will be a common name for all lengths commensural with that of the stick. Now if we apply this stick to another, which we will call A, and they be found to be commensura, we will say, A and 1 are commensura, $A=1$; and if then we apply the first stick to a third one which we will call B, and find them to be commensura, we will say, B and 1 are commensura, $B=1$; then we have the syllogism $A=1, B=1$, therefore $A=B$. And all the axioms of geometry are founded immediately upon the syllogistic process, though homon is at the bottom of the whole thing. If equals be added to equals, the sums will be equal, is very plainly founded on the syllogism. If $A=B$, as they are commensura, we may call each of them—three; and is $A'=B'$, as they are commensura, we may call each of them—two; then if we apply the homonical unit of measurement to A, we find that thrice one and A are commensura and so also of B; and if we apply the unit to A', we find that twice ONE and A' are commensura and so also with B'; then $A+A'$ must be equal to five times one, or FIVE, and $B+B'=$ five times one, or five; and we have the syllogism, $A+A'=5, B+B'=5$, therefore $A+A'=B+B'$. So also when we say that magnitudes, which being applied to each other coincide throughout their whole extent, are equal, this axiom is founded upon the syllogism: and in this case we come closer to the homon at the bottom. Suppose we have before us a certain object called A, and another called B; if now we represent the magnitude of A by a, and that of B by b, we must then say, the magnitude of A and a are homon, and the magnitude of B and b are homon; but if A and B cannot be discriminated otherwise than heterically, if they coincide, they are commensura, $a=b$, and each of them may be called d, and we have m. of $A=d$, m. of $B=d$, therefore m. of $A=m.$ of B. But if in the above case A and B can be incommensurated, we would have m. of A and a are homon, m. of B and b are homon: but $a < b$ therefore m. of A and a are homon, $a < b$, therefore m. of $A < b$; but m. of B and b are homon, m. of $A < b$, therefore m. of $A < m.$ of B, which is the foundation of the axiom, that The whole is greater than any of its parts. For, let m. of the whole be represented by A, and the m. of any part be represented by B; then m. of the whole and A are homon, m. of part and B are homon, but $b < a$, therefore etc.

Now it may be said, that it is strange that axioms which are regarded as sufficient truths should after all, be arrived at by a process so difficult to understand. This however, is not strange at all, the mind runs this course, as it were, in a flash and perceives the truths expressed by axioms without much difficulty; though to trace this course or process of the mind is very difficult. There are, however, some of the elementary truths in geometry, regarded as axioms, which seem, at first, to be peculiar, and they have been called inductions; they are contained in such propositions as the following; "Two straight lines, which have two points in common cannot afterwards diverge," "Two straight lines cannot inclose a space," "Two straight lines intersecting each other cannot both of them be parallel to a third straight line," and so on; Such propositions, however are founded upon the syllogism. Take the proposition, Two straight lines having two points in common cannot afterwards diverge, or what is equivalent, Two straight lines having two points in common must coincide throughout their whole extent. Now a mere glance at the proposition will show us that it is grammatically in the potential mode. And it must be evident that there are differential courses, i. e., that a straight course and a crooked one are differentia, and that two lines one of which runs a straight course and the other a crooked one, are in capacity differentia; given a certain number of points in space, a line that can run through all of the points, and a line, that can run through only some of them, are inter se differentia.

A B C B C

B

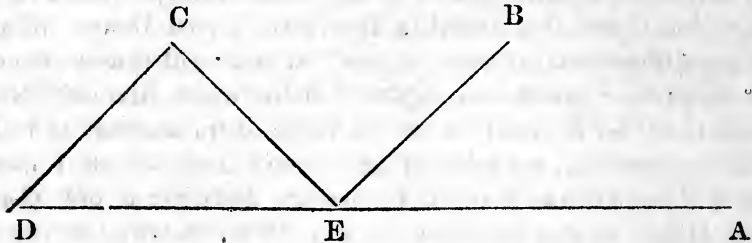
C

D

Let, therefore, A B C, be a straight line and let A represent its uniform course from A to C; then a straight line and A are homon. But let A B D be another line, whose course from A to D is represented by B, then A B D and b are homon. Then if A and B be homon, or similia, A B C and A B D will be similia. But the circumstance that the point at D in b can be discriminated from any point in the course A, and that at A and B, A and B coincide, shows that the courses A and B are differentia. Then the capacity of the line A B C, and A are homon, and the capacity of the line A B D and B are homon, but A and B are differentia; therefore we have, capacity of line A B C and A are homon, A and B are differentia, therefore, capacity of line A B C and B are differentia; but capacity of line A B D and B are homon, capacity of line A B C and B are differentia; therefore A B C and A B D are differentia; A B C however, by hypothesis, is a straight line, therefore A B D is not a

straight line. And from the foregoing demonstration, we can easily see, how the syllogism underlies the proposition that two straight lines cannot inclose a space; for, a space inclosed is a space surrounded by consecutive points, and if we lay down one straight line, another straight line touching the first one in any two points, cannot diverge from it, but must coincide with it in its whole course; but a course, a mere uniform direction can inclose nothing.

We have gone, we hope, far enough, to show that the axioms of mathematics are founded upon the syllogism, and leaving the axioms, therefore, we will give one illustration of the principles of our system of reasoning from a simple proposition in geometry. Take the proposition that the sum of the angles of a triangle are equal to two right angles.



Let D E C be the triangle and prolong the side DE to A; and from the point E draw EB parallel to DC, then from previous syllogisms we know that the angles CDE and BEA are commensura; we also that the angles CEB and DCE are commensura. But as CDE and BEA are commensura, we may call them by the common name A; and as CEB and DCE are commensura, we may call them by the common name B; Then either CDE or AEB is an A, and either CEB or DCE is a B; and we may call CED, C; then A B and C and the sum of the angles of the triangle are commensura. But the sum of all the angles that can be formed at a given point on one side of a straight line and two right angles are commensura, the angles A, B and C are the sum of the angles so formed at the point E, therefore A, B and C together, and two right angles are commensura.

CHAPTER XX.

ENTHYMEME, SORITES AND DELEMMA.

Having explained in the previous chapters the manner in which the syllogistic process proceeds, we do not deem it necessary to elaborate much upon the Enthymeme, Sorites or Delemma. When either one of the premises of a syllogism is expressed and the other understood, the expressed premise with the conclusion is called an enthymeme; as Iron will rust, therefore the plowshare will rust, or The plowshare is iron, and therefore it will rust. In such cases, it is easy to supply the premise, which is understood. Any per-

son, well grounded in the principles of the syllogism, will have no difficulty in managing the enthymeme.

Now when a conclusion has been legitimately drawn from premises, this conclusion may be made a premise and combined with either of the former premises, and another conclusion may then be drawn; and then this latter conclusion may be combined as a premise with the first and so on. When we continue to syllogize in this manner, the chain of syllogisms is called a Sorites; as, A and B are similia, B and C are similia, therefore A and C are similia; but C and D are similia, therefore A and D are similia; but D and E are similia, etc. And this process may be pursued with any of the modes and figures, which we have given in the preceding paradigms. Thus: A and B are similia, B and C are differentia, therefore A and C are differentia; but C and D are similia, therefore A and D are differentia; but D and E are differentia, therefore A and E are differentia or similia etc. Now we stated in a previous chapter that there are five objective nominal truths, and if we let A stand for one of them, B for another and C, D and E for the others severally, we may syllogize upon them in the following manner: A and B are hetera, B and C are hetera, therefore A and C are hetera; but C and D are hetera, therefore A and D are hetera; but D and E are hetera, therefore A and E are hetera; and therefore A, i. e., the thing distinguished by the name A, and taste, or sound, or feeling, or color, or scent are homon. And this shows us the manner in which we come to use disjunctive propositions; they are conclusions of syllogisms. The sky is either clear or cloudy, why? There are two states, capacial gregaria, of the atmosphere, distinguished inter se by the names clear and cloudy; one of these states now exists, therefore it is either clear or cloudy, i. e., the present state of the atmosphere and either clear or cloudy are homon. And when we say that Men are either black or white or tawny; this is a conclusional proposition drawn in the same manner as the one above: though there might be men of neither of these complexions, for aught we know. And in the conclusional proposition just given, that which is really affirmed is that one of the facial gregaria of every man and one of the three colors namely, black, white or tawny, are similia. And as similia have the same name, the color of any man and black or white or tawny are homon. Again: Iron and glass are hetera, hetera are divided into two classes, namely, similia and differentia, therefore iron and glass are either similia or differentia.

Now by the combination of disjunctive conclusional propositions in premises, we form the basis of what is called the Dilemma; thus, A and either B or C are similia, i. e., A and one of the two are similia, but either B or C, i. e., either one of them, and D are similia, therefore A and D are similia, therefore A and D are similia. And it must be noticed that there is an ambiguity in the use of the correllatives, EITHER, OR. In the first instance

A and either B or C are similia, we mean that A and one of the two are similia, while A and the other may be differentia for aught that is disclosed by the proposition; while in the latter instance, either B or C and D are similia, we mean that B and D are similia and also that C and D are similia. And it is this ambiguity in the use of the correllatives, that makes the dilemma kind of trap by which men are caught before they are aware of it.

Now if we set down before us the propositions, A and either B or C are homon, but either B or C and D are homon, and be careful not to be misled by the ambiguity of the correllatives, we can easily see by considering these propositions how we come by such hypothetical enthymemes as the following; if A and B are homon, A and D are similia, and if A and C are homon, A and D are similia; but A and B or C are homon, therefore A and D are similia; (mode 1st). But taking again the two disjunctive propositions A and either B or C are homon, and D and either B or C are homon, and taking the correllatives in both instances to mean one of the two and not the other, we will have for conclusion that A and D are either similia or differentia. If A and B are homon, and D and B are homon, A and D will be similia; and if A and C are homon, and D and C are homon, A and D will be similia; but if A and C are homon and D and B are homon or A and B are homon and D and C are homon, A and D may be differentia. Again; if we take the propositions A and either B or C are similia; E and neither B nor C are similia, it will follow that A and E are differentia.

Now it is evident that we may take any categorical proposition and put into a hypothetical form. Take the proposition, Ice is cold; and we may say, If ice is cold; but from this latter expression, we expect some conclusion to follow, and we state the proposition in this hypothetical manner for the purpose of drawing some conclusion, and therefore we give it this illative wording. And in such cases we always take one of the premises of a syllogism and state it hypothetically. Take the syllogism, Rainy weather is wet weather, it is rainy weather, therefore it is wet weather; now we may state the second premise hypothetically, If it is rainy weather, and draw the conclusion, It is wet weather, leaving the first premise unexpressed. We call such arguments hypothetical enthymemes; and those expressions of argument which have been commonly called hypothetical syllogisms, are merely hypothetical enthymemes stated first, and then throwing off the hypothesis, the enthymeme is stated again categorically, to show that the conclusion does not only follow logically, but also that the premises, from which the conclusion is drawn, are actual. Thus if A and B are similia then A and C are similia; but A and B are similia, therefore A and C are similia. In this example, the conclusion introduced by THEREFORE does not at all depend upon the expression, if A and B are similia, then A and C are similia, but upon, A and B are similia and another premise understood. In the syllo-

gism, A and B are similia, C and B are similia, therefore A and C are similia, any person, who looks at it, and grants that C and B are similia, will readily go farther, and grant that, if A and B are similia, if such be really the case, then A and C are similia, and when you convince him that really A and B are similia, the hypothesis is thrown off, and he acknowledges that A and C are similia. In such cases the conclusion is not drawn from the first hypothetical enthymeme—if A and B are similia then A and C are similia, but from A and B are similia and the other premise, B and C are similia, understood. If Socrates is virtuous, then he merits esteem; but Socrates is virtuous, therefore he merits esteem; why? The virtuous merit esteem. Socrates is virtuous therefore he merits esteem. We do not consider it necessary to go into an elaborate discussion upon such matter, we will, however, subjoin a note.

NOTE.—It is strange that neither Archbishops Whately nor Sir Wm. Hamilton were able to sound to the bottom of what they call hypothetical propositions, nor be able to perceive the true nature of the dilemma. "A hypothetical proposition," says Whately, "is defined to be two or more categoricals united by a copula (or conjunction) and the different kinds of hypothetical propositions are named from their respective conjunctions: viz: conditional disjunctive, causal &c." And again; "A conditional proposition has in it an illative force, i. e., it contains two and only two categorical propositions, whereof one results from the other (or follows from it)." And again, "A disjunctive proposition may consist of any member of categoricals." That a proposition may be the subject of another proposition is very clear; as that John is a scholar, is not denied; but that one proposition may be a dozen propositions is certainly very strange. Sir Wm. Hamilton adopting the explanation of Krug says (page 168) "Although, therefore, an hypothetical judgment appear double, and may be cut into two different judgments, it is nevertheless not a composite judgment. For it is realized through a simple act of thought, in which IF and THEN, the antecedent and consequent are thought at once and as inseparable. The proposition if B is, then A is, is tantamount to the proposition, A is through B. But this is as simple an act as if we categorically judged B is A, that is, B is under A. Of these two, neither the one—IF THE SUN SHINES,—nor the other,—THEN it is day—if thought apart from the other, will constitute a judgment, but only the two in conjunction." Now the above is a misconception of the nature of propositions, and it arises from the erroneous notions, entertained by Hamilton and others respecting predication. Suppose in the above example given, we leave the IF and THEN out, we will then have, the sun shines, it is day: Now Mr. Hamilton would admit that here are two propositions, but would answer that although there are two, yet it is not shown in any way that the one is connected or dependent upon the other without the words IF and THEN. Very well; take the propositions A is B, C is B, A is C, and without the word THEREFORE, before the last one, it is not shown by words, that this is the conclusion of a syllogism. And if the words IF and THEN possess the magic power to merge two propositions into one, we may use them also to merge a syllogism into a proposition, thus; if A is B and C is B then A is C, which according to Mr. Hamilton would be merely an hypothetical proposition. For when we say if A is B and C is B, we expect something to follow and

we perceive that A is C, does follow, and hence we may regard all this as but one continuous act of the mind. And respecting the expression "if A is, then B is, or A is through B," this expression is not true in any case excepting when A is the cause and always accompanied by the effect B. Mr. Hamilton's erroneous notions of what he calls an hypothetical proposition led him to misunderstand entirely, what he calls an hypothetical syllogism. On page 246, following Esser, Hamilton says, "If however, an hypothetical proposition involve only the thought of a single antecedent and of a single consequent, it will follow that any hypothetical syllogism consists not of more than three, but of less than three capital notions; and, in a rigorous sense, this is actually the case. On this ground, some logicians of great acuteness have viewed the hypothetical syllogism as a syllogism of two terms and of two propositions. This is, however, erroneous; for in an hypothetical syllogism, there are VIRTUALLY three terms. That under this form of reasoning, a whole syllogism can be envolved out of not more than two capital notions, depends on this, that the two constituent notions of an hypothetical syllogism present a character in the sumption altogether different from what they exhibit in the subsumption and conclusion. In the sumption these notions stand bound together in the relation of reason and consequent, without however, any determination in regard to the reality or unreality of one or the other; if one be, the other is, is all that is enounced. In the subsumption, on the other hand, the existence or non-existence of what one or the other of these notions comprises is expressly asserted, and thus the concept, expressly affirmed or expressly denied, manifestly obtains, in the subsumption, a wholly different significance from what it bore when only enounced as a condition of reality, or unreality, and in like manner, that notion which the subsumption left untouched, and concerning whose existence or non existence the conclusion decides, obtains a character altogether different in the end from what it presented in the beginning." This explanation Hamilton obtained from Esser. And hence from the above reasoning, if we suppose that we have before us a hat and a broom (which very supposition implies two separate existences) and we say, if the hat is not the broom, the hat and broom are are separate existences; but the hat is not the broom, therefore the two are separate existences, we make a sumption to get at some VIRTUAL third term, and this third term is envolved, because the terms in the sumption stand together in the relation of reason and consequent, and in the subsumption they they are asserted to be realities, and HENCE the third term; then in the subsumption we take the sumption to be actual and real, and by this method of syllogising, we prove that the hat is not the broom, just as we supposed in the sumption. It is astonishing that a man of so great learning and natural ability as Hamilton should have been drawn into this subtle and trifling nonsense of the German. In what Hamilton calls Dilemmated judgments, he and Whatley are also in the dark. Hamilton says (on p. 170) "Dilemmatic judgments are those, in which a condition is found, both in the subject and in the predicate, and as thus a combination of an hypothetical form and of a disjunctive form, they may also appropriately be denominated Hypothetico-disjunctive. If x is A, it (x) is either B or C—if an action be prohibited, it is prohibited either by natural or by positive law." * * * * * Now I apprehend, it will be impossible for any one to see why x is either B or C, granting that x is A, without going through the process—A is either B or C, x is A, and therefore x is either B or C. Hamilton carries his errors respecting Dilemmatic propositions into what he calls Dilemmatic syllogisms; but we will not criticise further.

CHAPTER XXI.

THE SINGULAR HOMONICAL SYLLOGISM.

Having treated of the Singular Syllogism and having explained pretty thoroughly the manner of the syllogistic process in the mind, it yet remains for us to show the further application of this process in the acquisition of knowledge. We have already shown that from the combination of two homonical propositions, as premises, we may gain similia or commensura in the conclusion. And if we represent aggregate existences by B, C, D, E &c., and any simple existence by A, we may then form an indefinite number of homonical propositions, all of which shall have A as the predicate, thus:

Therefore similia	{	Gregarium of B and A are homon,
Therefore similia	{	" " C and A are homon,
Therefore similia	{	" " D and A are homon,
Therefore similia	{	" " E and A are homon,
Therefore similia	{	" " F and A are homon.

And so on, which is a continued syllogism or Sorites. And a mere glance at the above chain of syllogisms will show us that, the SIMILE A, exists in B, C, D, E, &c.; and if B, C, D, E, &c., each points out an individual object, a swan, for instance, and A stand for white, for instance, we would say that Swan B, Swan C, Swan E, and all Swans which we have seen, are white; they are all similia in this facial gregarium. This is generalization from experience. And by the aid of conversation and books, we can of course, use the experience of others in the same manner as our own. And if from this experience we make an inference beyond experience, this process, which is wholly syllogistic within experience but no further, has been called by Bacon, "*inductio per enumerationem simplicem, ubi non reperitur instantia contradictoria.*"

But if we return to the nominal truths spoken of at the beginning of our inquiries, any person will readily admit that all colors, not only those, which have been seen, but also those which can be seen, are visible, i. e., visibility and a sine qua non of colors are homon. Color, however, is not an aggregate but a simple existence and therefore, not one of the gregaria of color, but color itself and visibility are objectively homon. And an objective homon, however often its times can be heterated, is nevertheless always homon; time in its modifications of past, present and future, cannot strengthen or weaken our belief in a homon. If I take a marble and inclose it in my hand for four hours, when I first put it into my hand I believe it to be an homonical thing, and at the end of four hours I believe it to be the homonical thing without doubt; the only thing, that can make me doubt of an objective homon, is that I do not always feel certain that, in the course of time, the homonical thing may not have been removed and a simile have been put into its place. The heteration of an objects times can have no heterating effect upon the object itself. The power of the mind to heterate, indeed, depends

upon time and space; but respecting the ego per se and the non ego per se, homon is homon irrespective of time. And hence if we take an object as time can have no heterating effect upon it, a thousand years from to-day, it will be homon; and although time has been personified and endowed with capacial gregaria by the poets, it must be evident that time per se has nothing in it, to produce any effect upon the ego or non-ego. But as time has no capacity to heterate or differentiate objects, if we affirm that this where is a where of pure space, i. e., this where and one where of pure space are homon, and that where and a where of pure space are homon, it must follow that this where and that where are similia. If time per se can neither heterate nor differentiate, any two wheres of pure space are now, always have been, and always will be, Similia, so long as pure space and pure space are homon; and so also with every other object so far as time per se is concerned.

But we may ask ourselves, has space any capacial gregaria to affect objects occupying it? And by the artificial production of a vacuum, we are able to decide upon reflection that here, in this instance, is a space, which has no capacity to interfere in any manner with objects occupying it, were any object in it. But if homon is homon, if space is space, this particular vacuated space and any other where of pure space are similia, they cannot be differentia, and hence no space can heterate, differentiate or incommensurate objects occupying it. We have therefore eliminated time and space, as agents, from our consideration; but before proceeding farther, we must explain some terms, which we will have occasion to use hereafter.

If we take any homon, this homon to-day, will be homon a thousand years hence, so far as time and space are concerned, we will, therefore, call this homon an homonical homon. But if we take another homon, a like case will be with it, and to distinguish the second homon from the first, we will call it an heterical homon; an homonical homon and an heterical homon will then be hetera.

Again; If the homonical homon and the heterical homon be inter se similia, we may call the heterical homon with reference to the homonical homon, a similical homon. An homonical homon and a similical homon will then be similia.

Again; If the homonical homon and the heterical homon be inter se differentia, we may call the heterical homon with reference to the homonical homon, a differential homon. An homonical homon and a differential homon will then be differentia.

Again; If the homonical homon and the heterical homon be commensura, we may call the heterical homon a commensural homop. An homonical homon and a commensural homon will then be commensura.

But again; If the homonical homon and the heterical homon be incommensura, we may call the heterical homon, an incommensural homon. An homonical homon and an incommensural homon will then be incommensura.

The following list will show the terms and the manner in which they distinguish objects:

- | | | | |
|------|--------------------------|---|---------------|
| 1st. | Homonical homon— a | } | homon. |
| | Homonical homon— a | | |
| 2d. | Homonical homon— a | } | hetera. |
| | Heterical homon— b | | |
| 3d. | Homonical homon— a | } | similia. |
| | Similical homon— a' | | |
| 4th. | Homonical homon— a | } | differentia. |
| | Differential homon— b | | |
| 5th. | Homonical homon— 2 | } | commensura. |
| | Commensural homon— $2'$ | | |
| 6th. | Homonical homon— 2 | } | incommensura. |
| | Incommensural homon— 3 | | |

Now with the above terms, the following syllogisms which we call singular homonical syllogisms, because one premise at least in each mode is homonical, may be constructed:

MODE 1ST.

The homonical homon A , in the place— B to-day, and the homonical homon— A , in any where a thousand years hence, are homon.

The homonical homon— A' , in the place— C to-day, and the homonical homon— A' , a thousand years hence in any where are homon.

Therefore the homonical homon— A , in any where a thousand years hence, and the homonical homon A' , in any where a thousand years hence, are similia.

MODE 2D.

The homonical homon— A , in the where B , to-day, and the homonical homon— A , in any where a thousand years hence, are homon.

The homonical homon— A , in the where, B to-day, and the heterical homon c , in the where— D to-day, are hetera.

Therefore the homonical homon— A , in any where a thousand years hence, and the heterical homon c , in any where a thousand years hence, are hetera.

MODE 3D.

The homonical homon A in the where B to-day, and the homonical homon A in any where, a thousand years hence are homon.

The homonical homon A in the where B to-day, and the similical homon A' in the where C to day are similia.

Therefore, the homonical homon A in any where a thousand years hence, and the similical homon A' in any where a thousand years hence, are similia.

MODE 4TH.

The homonical homon A in the where B to-day, and the homonical homon A in any where a thousand years hence are homon.

The homonical homon A in the where B to-day, and the differential homon C in the where D to-day are differentia.

Therefore the homonical homon A in any where a thousand years hence, and the differential homon C in any where a thousand years hence are differentia.

MODE 5TH.

The homonical homon A in the where B to-day, and the homonical homon A in any where a thousand years hence are homon.

The nomonical homon A in the where B to-day and the commensural homon A' in the where C to-day, are commensura.

Therefore the homonical homon A in any where a thousand years hence and the commensural homon A' in any where a thousand years hence are commensura.

MODE 6TH.

The homonical homon A in the where B to-day and the homonical homon A in any where a thousand years hence are homon.

The homonical homon A in the where to-day and the incommensural homon C in the where D to-day, are incommensura.

Therefore, the homonical homon A in any where a thousand years hence, and the incommensural homon C in any where a thousand years hence, are incommensura.

After a careful study of the above mode in the singular homonical syllogism, the following reasoning, we believe, will appear obvious. If we let a homon, always be homon in our minds, and we make this homon a SIMILE, i. e., if the homon A in the where B, have a simile in the where C, and another in the where D and so on, each one of these similia must have a common name, and no matter if their heterical number be infinite and the points of time of some be in the past, of others in the present, and of still others in the future, yet we have no hesitation in believing that each one must be an A. for if it should not be so, homon would not be homon; and that the really same thing should not be the same thing is absurd and impossible. But the homon in our minds has a simile in the minds of other men, and hence we believe without a doubt that two beings like ourselves a thousand years hence all colors, which they will know any thing about, will be visible, i. e., color and visibility will be to them homon. The same thing is the same thing, homon is homon, no matter about the modifications of time and space. Color and visibility are homon, visibility and visibility are homon. Therefore, color and visibility are similia (mode 1st) as the must be, if the visibility, in the first premise and that in the second be objectively heteral; and two objectively heterical existences, one in each premise, must always be found in the premises of every syllogism. And hence the general proposition that all colors are visible, is established beyond a doubt by the syllogistic process.

The proposition that all sounds are audible, or that sound is, has been, and ever will be audible, is established in the same manner. And thus we may deal with all the homonical propositions in which both the subject and predicate are the simple existances, which we have called facial gregaria. That all red is red, that all sweet is sweet, or that all white is, has been, and ever will be white to human beings, nobody doubts, because a contrary supposition is not only inconceivable but impossible, unless similia and differentia are homon.

Let us now turn our attention to capacial gregaria, and we will first notice figure or form. It is a proposition not worth discussing after what has already been said respecting homonical propositions and space, that every aggregate existence must have some figure or form. But were ten millions of forms inter se differentia known to our minds (and about things unknown we cannot reason) and we should give a name to distinguish any one figure or form, each other figure, which was a simile of the figure named must receive the name given to the homonical figure, which name now becomes a common name for all similia. If for instance we distinguish from other things any round ring by the name CIRCLE, then any round ring thus distinguished from other things, has been, is and always will be a circle. First round ring and A circle are homon, second round ring and first are similia. Therefore second round ring and A (one) circle are homon. And so also with squares, cubes, triangles, parallelograms, &c.

And it must be evident that if in any relation of parts, anything which may be called a quality, be found in any figure, this quality must have a simile in any other figure, which is a simile of the first figure. For, all the relations of the space inclosed by the outlines of any figure, must be inclosed in like manner by the outlines of all figures which are inter se similia. Certain points and their relations inter se in space constitute a figure, and when we lay down all the points and their relations, which that figure can contain. One circle and another are similia, the commensural relation of the diameter and one third of the circumference is in the first circle, i. e., the where of such commensural relation of points in space and the where of the points contained in the first circle are homon, therefore this commensural relation is in the second circle: and as circles are similia of space, this relational simile is in all circles at any time in any where. And such is the case with all the geometrical figures.

But if we consider the forms of animals, vegetables or minerals, we will find but few perfectly similia. The human form has no homonical standard by which to determine similia. If we should give certain and definite relations of points in space as the human form, we then might reason upon such human form with logical mathematical certainty, but our reasoning would only be approximately true when applied actually to individuals.

For the homonical standard which we have assumed, has not a simile in each individual of mankind; yet there is an approximation to similia in the forms of human beings sufficient usually to distinguish the human form from that of other animals. And this sufficient approximation to an homonical standard in one species, and the approximation to an homonical standard in another, enable us to affirm differentia anywhere, now, in time past, and in the future. We may say with all confidence that the form of any man and the form of any lizzard are, have been and always will be differentia. The human form, though not a simile of any homonical standard, is sufficiently distinguished by its relations from others; and were it not so, we could not tell the human form from others. We can not, indeed, point out any particular in the form of John, and say that wherever man is found, you will find a simile of this particular, but we can point out a number of particulars in John and say with confidence that wherever man is man there will be an approximation to these particulars. Of the forms of animals, vegetables and minerals then, we can not usually find an homonical type, and hence we can draw but approximate conclusions respecting the individuals which we have not seen.

Let us next consider impenetrability. We say and believe that all matter is impenetrable; and impenetrability being a simple existence and the predicate of an homonical proposition whose subject is an aggregate existence, we mean of course, that one of the gregaria sine qua non of matter and impenetrability are homon. And why do we believe this? Simply because we believe that homon is homon in any where at any time. Take the proposition All matter occupies space; and if this needs proof, we may take any piece of matter and we will see that this piece occupies space; we will see also that a where occupied and a where unoccupied are differentia; then the where of this piece of matter and an occupied where are homon; an unoccupied where and an occupied where are differentia; but if another piece of matter can exist in an unoccupied where, then the where of the first piece and the where of the second one are differentia. But all unoccupied wheres in space are similia, because space is space, homon is homon; the capacity to occupy, therefore in any where, is the only thing that can make an occupied where and an unoccupied where, differentia. But this capacial gregarium must reside in the thing occupying, and therefore matter having this gregarium and matter without it are differentia. But matter is matter, homon is homon, and this capacial gregarium is the sine qua non, which makes different pieces of matter similia, and therefore all matter must occupy space. Impenetrability in objects is nothing more than the capacity to remain in space; for, so long as an homonical object remains in space, the homonical where, in which it is, cannot be occupied by a heterical object, unless hetera and homon are homon, which is impossible. So long therefore, as matter is

matter, impenitrability will be its capacial gregarium. That, which has no where, cannot be matter, and hence matter, whose impenitrability has been destroyed, is no longer matter, it is no longer anything. The homonical where of an homonical existence called matter and AN (one) occupied where are homon, an occupied where and an unoccupied where are differentia; but if the homonical matter in the homonical where be destroyed by heterical matter, the homonical where cannot be occupied by the homonical matter unless hetera and homon are homon, which is impossible; therefore any matter must occupy space. This, however, does not prove that matter cannot be annihilated; it only proves that wherever and whenever, matter is matter, it will occupy space; that all matter is impenitrable. Whether matter can be annihilated or not, we have no data from which we can decide the question. Water may be inclosed in a golden ball and pressed through the gold, but this only proves that by such means matter cannot be annihilated.

The use of the syllogistic process in establishing a *sine qua non*, by the SINGULAR HOMONICAL SYLLOGISM, which we have just been discussing, seems to be what J. Stuart Mill considers the true type of INDUCTION, when he defines induction to be "the operation of discovering and proving general propositions." Mr. Mill, however, like all other writers upon induction, seems to have had no definite conception of the thing for which he was on the lookout, and he would not have been able to have identified it, if he had found it. In one place induction is "the operation of discovering and proving general propositions;" in an other it is "generalization from experience;" in an other it is "that operation of the mind by which we infer that what we know to be true in a particular case or cases, will be true in all cases which resemble the former in certain assignable respects;" and again, "to ascertain what are the laws of causation which exist in nature; to determine the effects of every cause, and the causes of all effects, is the main business of induction; and to point out how this is done is the chief object of inductive logic." Mr. Mill is an able writer, but his logical induction is, in a great measure, an ignus fatuus.

CHAPTER XXII.

THE PLURAL HOMONICAL SYLLOGISM.

Having shown in the last chapter how we generalize from experience, and also how in certain cases we may select a simple homonical existence and prove it to be a *sine qua non* by the singular homonical syllogism, we must pursue the syllogistic process still further and show how we reason by the Plural Homonical Syllogism. If we put two balls before us, we will say that they are hetera, i. e., that the one is not the other; if, however, we turn our eyes away from them for a few moments, or cover them with our hand, and then we remove it from them and look at them again, we will say,

that they are the SAME balls. But by this expression we do not mean that the one and the other are homon, for we know that inter se they are hetera, but what we really mean, is, that the two balls under our eyes THEN are the identical balls under our eyes NOW, i. e., the two balls THEN and the two balls NOW are homonical hetera. And before proceeding further, we must again explain some terms, which we will have occasion to use in our future inquiries.

We have already seen that time and space per se have no capacity to heterate, differentiate or incommensurate objects in time and space, that subjectively two, but objectively one homonical ball to-day, so far as time and space per se are concerned, will be objectively homon, to-morrow and forever; we will therefore call such homa, homonical homa. But if we take subjectively two other balls, which are objectively homon, they are related to themselves in like manner as the first two, we will call them heterical homa: Homonical homa and heterical homa will then be hetera.

Again, If two homonical hetera be inter se hetera to-day, so far as time and space are concerned, they will remain hetera, and therefore we will call them homonical hetera; but if we take two other hetera, they also will remain hetera, and to distinguish them from the first two, we will call them heterical hetera: Homonical hetera and heterical hetera will then be hetera.

Again, If two homonical hetera be inter se similia to-day, so far as time and space are concerned, they will remain similia inter se, and therefore we will call them homonical similia; but if we take two heterical hetera inter se similia, they also will remain inter se similia, and to distinguish them from the first two, we will call them heterical similia: Homonical similia and heterical similia will then be hetera.

Again, If we take two homonical hetera inter se differentia, they will remain differentia, and we will call them homonical differentia; but if we take two other hetera inter se differentia, they also will remain differentia, and to distinguish them from the first two, we will call them heterical differentia: Homonical differentia and heterical differentia will then be hetera.

Again, If we take two homonical hetera inter se commensura, they will remain inter se commensura, and we will call them homonical commensura; but if we take two other hetera inter se commensura, a like case will be with them, and to distinguish them from the first two, we will call them heterical commensura: Homonical commensura and heterical commensura will then be hetera.

Again, If we take two homonical hetera inter se incommensura, they will remain incommensura, and we will call them homonical incommensura; but if we take two other hetera inter se incommensura, a like case will be with them, and to distinguish them from the first two, we will call them

heterical incommensura; Homonical incommensura and heterical incommensura will then be hetera.

Again, If we take two hetera inter se similia, they will remain inter se similia, and we will call them homonical similia; but if now we take two heterical similia, and the homonical similia and heterical similia be inter se similia, to distinguish the heterical similia, we will call them similical similia: Homonical similia and similical similia will then be similia.

Again, If we take two homonical similia and two heterical similia, and the homonical similia and heterical similia be inter se differentia, we will call the latter differential similia: Homonical similia and differential similia will then be differentia.

Again, If we take two homonical differentia and two heterical differentia, and the one of the homonical differentia and one of the heterical differentia be inter se similia, and the other of the homonical differentia and the other of the heterical differentia be inter se similia, we will call such heterical differentia, similical differentia: Homonical differentia and similical differentia will then be similia.

Again, If we take two homonical differentia and two heterical differentia, and the homonical differentia and heterical differentia be inter se differentia, we will call such heterical differentia, differential differentia: Homonical differentia and differential differentia will then be differentia.

Again, If we take two homonical commensura and two heterical commensura, and they be inter se commensura, we will call the latter commensura, commensural commensura: Homonical commensura and commensural commensura will then be commensura.

Again, If we take two homonical commensura and two heterical commensura, and they be inter se incommensura, we will call the latter, incommensural commensura: Homonical commensura and incommensural commensura will then be incommensura.

Again, If we take two homonical incommensura and two heterical incommensura, and the one of the homonical incommensura and one of the heterical incommensura be inter se commensura, and the other of the homonical incommensura and the other of the heterical incommensura be inter se commensura, we will call the heterical incommensura, commensural incommensura: Homonical incommensura and commensural incommensura will then be commensura.

Again, If we take two homonical incommensura and two heterical incommensura, and they be inter se incommensura, we will call the latter, incommensural incommensura: Homonical incommensura and incommensural incommensura will then be incommensura.

The following list will show the terms and their relations:

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|--|---|
| 1. Homonical homa a.a. } homa. | 9. Homonical similia a.a. } differentia. |
| 2. Homonical homa a.a. } hetera. | 10. Homonical differentia a.b. } similia |
| 3. Heterical homa a.'a.' } hetera. | 11. Similical differentia a.'b.' } differentia. |
| 4. Homonical hetera a.a.' } hetera. | 12. Homonical differ. a.b. } commensura |
| 5. Heterical hetera b.b.' } hetera. | 13. Differential differ. c.d. } incommensura. |
| 6. Homonical similia a.a. } hetera. | 14. Homonical com. 2.2. } commensura |
| 7. Heterical similia a.'a.' } hetera. | 15. Com'sural com. 2'2' } incommensura. |
| 8. Homonical differentia a.b. } hetera. | |
| 9. Heterical differentia c.d. } hetera. | |
| 10. Homonical comensura 2.2. } hetera. | |
| 11. Heterical commensura 3.3. } hetera. | |
| 12. Homonical incomens'a 2.3. } hetera. | |
| 13. Heterical incom'ensura 3.4 } hetera. | |
| 14. Homonical similia a.a. } similia. | |
| 15. Similical similia a.'a.' } similia. | |

Now the following paradigms will show the syllogisms, which may be constructed with the foregoing terms, which syllogisms, as they have one homonical premise at least in each mode, we call plural homonical syllogisms.

Mode First.—The homonical homa a.a. to-day, and the homonical homa a.a. a thousand years hence, are homonical homa; The homonical a.'a' to-day, and the homonical homa a.'a.' a thousand years hence are homonical homa; Therefore the homonical homa a.a. a thousand years hence and the heterical homa a.'a.' a thousand years hence, are similical homa.

Mode Second.—The homonical homa a.a. to-day, and the homonical homa a.a. a thousand years hence, are homa; The heterical homa a.'a.' to-day, and the homonical homa a.a. to-day, are heterical homa; Therefore, the homonical homa a.a. a thousand years hence, and the heterical homa a.'a.' a thousand years hence, are heterical homa.

Mode Third.—The homonical hetera a.'a' to-day, and the homonical hetera a.'a. a thousand years hence, are homonical hetera; The homonical hetera a.'a. to-day, and the heterical hetera b.b. to-day, are heterical hetera; Therefore, the homonical hetera a.'a. a thousand years hence, and the heterical hetera b.b. a thousand years hence, are heterical hetera.

Mode Fourth.—The homonical similia a.a. to-day, and the homonical similia a.a. a thousand years hence, are homonical similia; The homonical similia a.a. to-day, and the heterical similia a.'a.' to-day, are heterical similia; Therefore the homonical similia a.a. a thousand years hence, and the heterical similia a.'a.' a thousand years hence are heterical similia.

Mode Fifth.—The homonical differentia a.b. to-day, and the homonical differentia a.b. a thousand years hence are homonical differentia; The homonical differentia a.b. to-day, and the heterical differentia c.d. to-day are heterical differentia; Therefore the homonical differentia a.b. a thousand years hence, and the heterical differentia c.d. a thousand years hence, are heterical differentia.

Mode Sixth.—The homonical commensura 2.2. to-day, and the homonical commensura 2.2. a thousand years hence, are homonical commensura; The homonical commensura 2.2. to-day, and the heterical commensura 3.3. to-day are heterical commensura; Therefore the homonical commensura 2.2. a thousand years hence, and the heterical commensura 3.3. a thousand years hence, are heterical commensura.

Mode Seventh.—The homonical incommensura 2.3. to-day, and the homonical incommensura 2.3. a thousand years hence are homonical incommensura; The homonical incommensura 2.3. to-day, and the heterical incommensura 4.5. to-day are heterical incommensura; Therefore the homonical incommensura 2.3. a thousand years hence, and the heterical incommensura 4.5. a thousand years hence, are heterical incommensura.

Mode Eighth.—The homonical similia a.a. to-day, and the homonical similia a.a. a thousand years hence, are homonical similia; The similical similia a.'a.' to-day and the homonical similia a.a. to-day are similical similia; Therefore the homonical similia a.a. a thousand years hence, and the similical similia a.'a.' a thousand years hence are similical similia.

Mode Ninth.—The homonical similia a.a. to-day, and the homonical similia a.a. a thousand years hence are homonical similia; The homonical similia a.a. to-day, and the differential similia b.b. to-day, are differential similia; Therefore the homonical similia a.a. a thousand years hence and the differential similia b.b. a thousand years hence are differential similia.

Mode Tenth.—The homonical differentia a.b. to-day and the homonical differentia a.b. a thousand years hence are homonical differentia; The homonical differentia a.b. to-day, and the similical differentia a.'b.' to-day are similical differentia; Therefore the homonical differentia a.b. a thousand years hence, and the similical differentia a.'b.' a thousand years hence, are similical differentia.

Mode Eleventh.—The homonical differentia a.b. to-day, and the homonical differentia a.b. a thousand years hence are homonical differentia; The differential differentia c.d. to-day, and the homonical differentia a.b. to-day are differential differentia; Therefore the differential differentia c.d. a thousand years hence, and the homonical differentia a.b. a thousand years hence are differential differentia.

Mode Twelfth.—The homonical commensura 2.2. to-day, and the homonical commensura 2.2. a thousand years hence, are homonical commensura; The commensural commensura 2.'2.' to-day, and the homonical commensura 2.2. to-day, are commensural commensura; Therefore the commensural commensura 2.'2.' a thousand years hence, and the homonical commensura 2.2. a thousand years hence, are commensural commensura.

Mode Thirteenth.—The homonical commensura 2.2. to-day, and the homonical commensura 2.2. a thousand years hence are homonical commensura; The incommensural commensura 3.3. to-day, and the homonical commensura 2.2. to-day, are incommensural commensura; Therefore the incommensural commensura 3.3. a thousand years hence, and the homonical commensura 2.2. a thousand years hence, are incommensural commensura.

Mode Fourteenth.—The homonical incommensura 2.3. to-day, and the homonical incommensura 2.3. a thousand years hence, are homonical incommensura; The commensural incommensura 2.'3.' to-day, and the homonical incommensura 2.3. to-day, are commensural incommensura; Therefore the commensural incommensura 2.'3.' a thousand years hence, and the homopi-

cal incommensura 2.3. a thousand years hence, are commensural incommensura.

Mode Fifteenth.—The homonical incommensura 2.3. to-day, and the homonical incommensura 2.3. a thousand years hence, are homonical incommensura; The incommensural incommensura 5.6. to-day, and the homonical incommensura 2.3. to-day, are incommensural incommensura; Therefore the incommensural incommensura 5.6. a thousand years hence, and the homonical incommensura 2.3. a thousand years hence, are incommensural incommensura.

If the reader has carefully studied what we have called the singular homonical syllogism in the preceeding chapter, the plural homonical syllogism will not need to be specifically explained. And any person can see that we are not necessarily limited to two homa or hetera; we may take the homonical homa or hetera a, b, c, d, e, &c., and deal with them in like manner as we have dealt with two homa.

Now if we take any simple existence in nature, any one will allow that this simple existence and itself are homon; and any one will agree also that so long as this simple existence and itself are homon, it and itself can not be hetera, and consequently it can not be a SIMILE of itself, nor can it and itself be differentia. And in a previous chapter we have shown that, when we look upon nature, we gain our knowledge of cause, in the first instance through effects, which are manifested by changes. And from what we have said already, it must appear, that a homon per se can not change: whatever it may be, so long as it exists, it is the homonical homon. If then we take any sine qua non, impenitrability for instance, this sine qua non is impenitrability to-day, always has been and always will be, homon is homon.

Now if we place before us an ivory ball, we have no doubts in affirming that one of the capacial gregaria sine qua non of this ball and impenetrability are homon; and if we put before us another ivory ball, we will make a like affirmation respecting it, and therefore the first and second balls are similia. And if the first gregarium be located in the homonical WHERE B, and the second one enter the homonical WHERE B, the first one must take an heterical where. For, in the respect of impenetrability the two balls are similia; and therefore the homonical similia a.'a. to-day, the one (a) in the homonical WHERE B, and the other (a') in the where C, and the homonical similia a.'a. to-morrow in any where are homonical similia. But respecting the homonical similia a.'a. to-morrow, if the second (a') be in the where B, i. e., if the where B occupied to-day by the first (a) to-morrow be occupied by the second (a'), the second (a') must have a SIMILE in the first (a), and the where of this SIMILE, and the where B must be hetera. But if (a') the first sine qua non be displaced necessarily from the where B by the entrance of the second (a) sine qua non, is not what has happened in a single instance sufficient to establish beyond a doubt that, whenever any WHERE is occupied by an A and another a' enters this where, the first a must be displaced? So long as homa are homa, this must be the case in any part of space at any point of time.

And if this be the case with the homonical similia a, 'a., must it not always be the case with all similical similia? And if we call this displacement of one impenetrable object by an other, a LAW, it must be evident that this law is uniform, i. e., this law and an uniformity are homon. And in a like manner we might treat of elasticity, of fluidity, of rigidity, lubricity and so on. And so long as homa are homa and similia are similia, we can not doubt of the uniformities in all instances.

But again if we take two differentia, oxygen and hydrogen for instance, we may reason upon them in like manner and with perfect exactness. For, oxygen being an elementary thing, so long as oxygen is oxygen, as homon is homon, any particular oxygen will contain all the gregaria of any oxygen, i. e., each gregarium of a particular oxygen will have a simile in any and every other oxygen: and so also with hydrogen. And hence if any homonical process unite them into water in any instance, a simile of this process will unite them into water in every instance. So long as homa are homa, similia similia and differentia differentia, we can not doubt that a result brought out of the homonical differentia a.b. by the homonical process d, will have a simile of that result brought out of the similical differentia a. 'b. by d', a simile of the homonical process d. And hence we must conclude that The laws of nature are uniform; is a proposition which is established in our minds by the syllogistic process. The result of the homonical differentia a.b. by the homonical process d and A are homon: The homonical differentia a.b. with the homonical process d and the similical differentia a. 'b.' with the similical process d' are similia; Therefore the result of the similical differentia a. 'b.' by the similical process d' and a are similia.

We have now said all that we deem necessary to be said at present while treating of the syllogism. We have given the syllogistic process a much more thorough analysis than it has received heretofore by writers upon logic, and we hope that our labors thus far will enable philosophers who shall come after us to see clearly the manner, application and use of the syllogism. We, however, must proceed further, and treat of induction, a subject, which, we are confident, has not been understood by writers upon that subject. Induction, therefore, will occupy our attention in Book II.

BOOK II.

CHAPTER I.

MISNAMED INDUCTIONS.

The processes of the mind concerned in induction, in our apprehension, have not been understood by any writer upon logic, with whose works we are acquainted. Bacon is said to have been the author of the inductive philosophy; but his *Novum Organum* shows the necessity of such a philosophy scientifically constructed rather than the actual construction in a methodical manner. His remarks, as far as they go, are not systematically arranged, and therefore they are often obscure; and from this reason with others, his suggestions, though frequently of the greatest importance, have not led his successors to glean from his aphorisms the true principles of induction and to work them into a scientific and methodical system of inductive logic. That Bacon had in view a better and greater system of philosophy than subsequent writers have made out of it seems to me to be certain. The aids for the understanding, about which he speaks so frequently, are suggested here and there in the second book of the *Organum*, but without any scientific theory to cement and make his remarks understood. History and experiments, without the knowledge of the inductive processes and their application can not aid the understanding in gaining certain knowledge of nature's laws; and these processes, as far as treated of, are not brought out in a scientific manner in the *Organum*. Men have always had nature before them but the method of interrogating her has not been understood. And though Bacon made a grand beginning at explaining this method, yet most subsequent writers have not only, not improved upon Bacon's work, but have underrated the value of such method.

There is no subject about which more erroneous notions prevail among philosophers, than about the subject of the inductive processes themselves; and these notions, in our opinion, are grounded upon erroneous notions about the syllogism. Philosophers are not at all agreed, about what processes, when pointed out shall be called inductive; and hence results, which are entirely owing to the syllogism, are often claimed as inductions, induction having some vague and unexplained meaning. The better way, however, to show what results are owing to the syllogistic process, is to explain the syllogism, and then the reader himself can make the application to any case, which may arise; this we have endeavored to do heretofore. And the better way to show what results are owing to the inductive processes, will be to explain these processes. But before doing this, from the manner in which the subject has been treated by authors heretofore, it is necessary, in order to be well understood by the reader, for us to show some things, which have

been called induction, but which in our system do not at all come under the meaning, which we attach to that term.

Archbishop Whately has treated of induction, but his erroneous notions, as we conceive, of the syllogism, led him to misconceive the nature of the inductive processes; though many of his remarks are valuable in helping to clear the way for a better understanding of the matter. The scholar, however, who has done more, perhaps, than any other, in clearing the way, is I. Stuart Mill. His treatise upon logic is learned and able, though we can not agree with him either upon ratiocination or induction. Archbishop Whately has well remarked that the syllogistic process is not the sole process necessary in reasoning in a syllogistic manner; and we may state that we do not consider the syllogistic and inductive processes together to be the only processes used in gaining truth, as any one will understand, who has studied the remarks made in the chapters previous to those treating of propositions and the syllogism in book 1. But to attempt to notice all the processes, which have been brought forward as inductive, but which we do not regard as such, would require too much room in this book, and besides, as we think, it will be unnecessary.

And first, when a name stands for, or points out a *sine qua non*, which distinguishes the existence for which it stands from others, we do not consider that the inductive process has anything to do with proving this *sine qua non*, or with proving the general proposition, which may be constructed upon this *sine qua non*. All those truths, which we have called nominal truths, are each of them, a *sine qua non* of themselves; and hence there is no induction in establishing the truth of the proposition that, every color, in any place at any time, is a color, but the truth of such proposition is established by the singular hemonical syllogism, as we have shown heretofore. Neither do we consider induction to be the collecting of a sufficient number of instances to warrant us in believing that the instances, which we have seen, are fair specimens of the class. We should think strangely of a man, who, after having been informed that the name island distinguishes a portion of land entirely surrounded by water should start on a tour to examine this and that island, until he had a sufficient number of instances collected to warrant the inference that, all islands are surrounded by water; yet Archbishop Whately's conception of induction does not rise higher than this. The Archbishop agrees with Aldrick, that, from the examination of this and that magnet, we conclude that all magnets attract iron; when in truth, magnetism, the quality of attracting iron, is the *sine qua non* of magnets, and it must of necessity exist in every thing, which may be called a magnet. And we dissent altogether from Mr. Mills definition that, "induction may be considered the operation of discovering and proving general propositions." And instead of believing with Mr. Mill, that induction is at the foundation of all

general propositions, we do not think that any general proposition can be established by induction. We therefore state to the reader that the process, about which we shall speak hereafter under the name of inductive, has nothing to do with establishing general propositions, and that such notion has a tendency to obscure the whole subject.

We must also be careful to avoid another error of Mr. Mill, in considering induction to be generalization from experience. We have heretofore shown that, generalization from experience proceeds upon the singular syllogistic process; and if we go any farther than experience and infer that cases to which mankind's experience does not extend, will be similia of those falling within that experience, the experience is not an inductive, but a probable one. The case given by Mr. Mill himself of the mistake made by mankind in inferring that all swans are white because they had seen a great number of white swans, and not a single instance of a swan of any other color, shows that the induction, if it be called so, was faulty, and in our estimation it was no induction at all, but merely a probable inference from numbers, the *inductio per enumerationem simplicem* of Bacon. Probable inferences may be drawn, with which we are perfectly satisfied, though we can not know that they are certainly true. Day and night have succeeded each other with perfect regularity so far as the experience of mankind extends, and for that reason alone there is a strong probability if we can see no cause for a change that such will be the case hereafter. But from the circumstances that no exception to a certain uniformity has fallen within the experience of mankind, we do not infer by the inductive process that there will be no exception hereafter. From the continuous uniformity, extending through experience, we are led to believe upon the ground of probability that the causes producing such uniformity will continue to act without interruption, though we know not what these causes are, nor that they will certainly continue uninterrupted.

The case of the naturalist inferring that all horned animals are cloven footed, because all those horned animals, which have fallen within the experience of mankind, are so, rests entirely upon probabilities, and not upon induction unless the *inductio per enumerationem simplicem* be true induction. If it had always happened within our experience that every Friday brought some ill-luck, the inference that every Friday in the future will be unlucky, would be just as probable to our minds as the case of animals with horns having cloven feet, yet there is nothing in the nature of such inference that corresponds to what we mean by induction.

Again, we do not agree with Mr. Mill in the office of induction in ascertaining the distance from the earth to the moon. Mr. Mill says, "the share which direct observation had in the work consisted in ascertaining at one and the same instant, the zenith distances of the moon, as seen from two

oints very remote from one another on the earth's surface. The ascertainment of these angular distances ascertained their supplements; and since the angle at the earth's centre subtended by the distance between the two places of observation was deducible by sperical trigonometry from the latitude and longitude of those places, the angle at the moon subtended by the same line became the fourth angle of a quadrilateral of which the other three angles were known. The four angles being thus ascertained, and two sides of the quadrelateral being radii of the earth; the two remaining sides and the diagonal, or in other words, the moon's distance from the two places of observation and from the center of the earth, could be ascertained, at least in terms of the earth's radius, from elementary theories of geometry. At each step in this demonstration we take in a new INDUCTION represented in the aggregate of its results, by a general proposition." Now we do not consider that there has been any INDUCTION at all in the above problem, but that, after the observations are made, the whole process is syllogestic; and any one, who has mastered what we have said heretofore in Book 1st, we apprehend, can make the application and demonstrate the problem by the syllogism.

Neither do we agree with Mr. Mill that the uniformity in the course of nature, or what is the same thing more definitely expressed that like causes with like conditions will produce like effects in any place at any time is the highest induction, nor do we consider it to be any induction at all. Neither do we consider "this assumption," to be as an assumption involved in any case of induction; nor can we consult the actual course of nature in this regard any farther than our experience extends, which is not sufficient to warrant an inductive influence. But we have shown heretofore that the uniformity of nature, or that like causes with like conditions will produce like effects in any place at any time, is demonstrated to our minds by the plural homonical syllogism.

There in an other improper use of the term, induction well pointed out by Mr. Mill, it is the case of the navigator approaching land and being at first unable to determine whether it be a continent or an island; but after having coasted around and having arrived at the same point from which he started he pronounces it to be an island. This navigator by connecting together all his observations finds that this land is surrounded by water, and every island is a portion of land surrounded by water, and therefore this land and islands are similia—this land is AN island. Mr. Mill continues to show that Kepler ascertained the figure of the orbit travelled by the planet Mars, by observations separately made but connected together in a like manner with the navigator, and justly concludes that there was no induction in the process. But Mr. Mill considers that Kepler did make one inductive inference, when he inferred that the planet would continue to revolve in an ellipse. Now if this inference was made upon the grounds that like causes

will produce like effects then the inference was syllogistic; but if it was made upon the grounds that the planet had always gone in an ellipse heretofore, the inference was a probable one; and in no case could such inference be made by induction.

These remarks might be continued at great length; but if the student has mastered the syllogism, he will be able to see that many results purely syllogistic have been attributed by authors to induction, and that the term induction is very often used without any definite meaning at all. Having, therefore, set the mind of the reader free, as we hope, so that he will not look in wrong directions, we will proceed and come nearer to the subject, and explain what we consider to be true induction.

CHAPTER II.

INDUCTION DISTINGUISHED.

Having spoken in the previous chapter of certain notions of induction which we wish the reader to keep out of his mind, while following us in our future inquiries, it seems necessary now to state what we mean by induction, as well as words can express our meaning in brief, and to give the reader some clue to the directions in which we propose to go in search of truth. Induction, then, is the result of those processes of the mind by which the unknown causes of any given effect are discovered; and the processes of the mind engaged in such discoveries are the inductive processes. We stated in a former chapter that we gain our knowledge of cause, in the first instance, through effect, i. e., we can not look upon any aggregate existence, and before we have the knowledge of effects, determine such existence to be or to contain a potential cause of any given effect. And in studying the inductive processes we must always have some given effect before our mind and from it determine the causes: the inductive processes have nothing to do in taking causes and from them determining effects. If indeed we take two elementary substances and put them together and a certain effect follow we take this effect and determine that those elementary substances were the causes of it; and when we have done so, we have also, from the correlative natures of cause and effect, determined that the phenomenon which we call an effect, is the effect of those causes; but we must always keep the effect in view, it must be in view always before the inductive processes can have any thing upon which to operate, while the causes of a given effect may be and always are entirely out of sight or without our knowledge when the inductive processes commence to search for them. If the reader will bear this in mind it will free the subject from much obscurity, which otherwise surrounds it.

And since cause and effect are always involved by the inductive processes, it is necessary also, to put the reader upon his guard that he may not

confound what are called *a priori* and *a posteriori* reasonings with induction. After that we have gained the knowledge of certain effects and their causes, we look upon these causes and their conditions, and infer, by the plural homonical syllogism, what effects will follow, without waiting to witness such effects by our senses. For instance, if a cannon be loaded with dry powder and a man be about to apply a match to it, by the plural homonical syllogism we infer that there will be an explosion. This application of the syllogism when we have the conditions as the homonical similia or differentia, whose effects we know in the premises, and from them we infer the effects of similical similia or differentia, whose effects have not yet transpired in time and space, is called *a priori* reasoning, or reasoning from cause to effect. Induction, however, has nothing to do with it.

On the other hand, if we see a cannon and hear the report of its discharge and we be asked, what is the cause of this report, from our former knowledge of such effect and its causes, by the plural homonical syllogism, we infer the cause of this particular effect. And this application of the syllogism, when we have an effect whose causes and conditions we know as the homonical homon in the premises, and we infer the causes and conditions of a similical homon, whose causes and conditions are not witnessed by our senses, is called *a posteriori* reasoning, or reasoning from effect to cause. But there is no induction in it.

Both *A PRIORI* and *a posteriori* reasonings are entirely syllogistic. In both, some particular case has brought by induction the knowledge, in the first instance, of a certain effect and the causes and conditions of it to our minds, and then this case furnishes the premises for the plural homonical syllogism to work with, either *A PRIORI* or *A POSTERIORI*. But when the causes of an effect, an homonical homon, are unknown, no inference can be drawn *a posteriori* respecting the causes of a similical homon; neither can any inference be made *A PRIORI* respecting the effect of similical similia or differentia, when the effect of homonical similia or differentia, the effect of such causes, is unknown to us. The knowledge of certain effects with their causes is already in the mind before *A POSTERIORI* or *a priori* reasonings begins. The inductive-processes take hold of any given effect, the causes of any similical effect and also of this given effect being without our knowledge, and search out and induct the conditions and causes of the given effect. Keeping, then, in mind that, *A PRIORI* and *a posteriori* reasoning are syllogistic and that they proceed from certain known cases to infer respecting similical cases, while the inductive processes proceed from a given phenomenon to make known to us the causes and conditions of that phenomenon, we will proceed farther, hoping that we will not be misunderstood.

Now in speaking of cause and effect, it is usual with philosophers to call the cause an antecedent and the effect the consequent. These terms,

antecedent and consequent, have reference to the relations of points in time, and we have already shown heretofore, that time possesses no capacial gregaria and that it can not be the cause of any change or effect. And therefore if we say that a cause is an antecedent, we must mean only that the existence whatever it may be, to which we refer as cause, occupied a point in time prior to the point occupied by the existence which we call the effect. And although this be true, yet it does not with any definiteness determine a cause. If we say that a cause is an antecedent of an effect, i. e., a cause and an antecedent are homon, we may still enquire what antecedent is referred to as connected with any given effect, for there are many things which existed in nature prior to John Smith's being intoxicated, and which antecedent is connected with this effect? It is, therefore, merely the condition of a cause that it exist antecedently to an effect; but antecedent is a term which can not be used as synonymous with cause.

Now we have shown heretofore that time and space can not be the causes of any thing; they are however the conditions of all causes and effects, and they are the only things of which we shall speak in our future inquiries as conditions. Every thing in nature which can be a cause, can be cause only upon the conditions of time and space, and that which has once been a cause, will in like conditions of time and space in the future be a cause again. A condition which presents the absence of a preventing cause is sometimes confounded with a cause, as the absence of the air in a pump is, sometimes said to be the cause of the water rising in it. This however is but a condition of space.

We define causes therefore, to be the capacial gregaria of the aggregate existences from which given changes or effects spring. To go behind the capacial gregaria of aggregate existences and inquire into the ontology of these capacial gregaria is no part of our undertaking at present. We may say, indeed, that they are the manifestations of the Deity's will, i. e., that they are the capacial gregaria of the Almighty himself made tangible to us; but we take these capacial gregaria of existences made known to us, as the only causes of which we shall treat and we shall regard them as the primary causes of all the effects in nature. If any one shall say that the Almighty is still a prior cause, we have no objection.

And it will occur to almost any one, after what has been said about cause and effect in a previous chapter, that an homonical capacial gregarium per se can not be a cause of any given effect; there must be heterical gregaria implicated before any effect can be produced. If we take an ivory ball, which possesses the capacial gregarium of impenitrability, i. e., the power to remain in space, we must see that this capacial gregarium per se can produce no effect whatever. If the ball be at rest its impenitrability can not start it, and if it be in motion its impenitrability can not stop it; the impeni

tr.ability in the ball can per se produce nothing. But if an other ball possessing also imperitrability be brought to bear upon the first one, the heterical impenitrabilities, one in each ball, can inter se produce an effect. Homon per se must always remain homon, and per se no effect can spring from an homonical gregarium; and hence all effects in nature are produced, not by an homonical gregaridm, but by heterical gregaria. And as the capacial gregaria of the aggregate existences, from which changes spring, are the causes of all the phenomena of nature, it is necessary in seeking for these causal gregaria of any effect, to find, in the first place, the aggregate existences possessing the said gregaria, and to sepearate them from others.

And in contemplating the aggregate existences, whose capacial gregaria are causes, it will readily appear that aggregate existences may be divided into primary, secondary, tertiary and quartuary aggregations. By a primary aggregate existence then, we mean what is usually called an elementary substance, and by secondary aggregations, those substances compounded of two elements, by tertiary aggregations, substances compounded of three elements, if such compounds exist in nature, and so on. And if we take any primary aggregate existance, a jar of oxygen for instance, as this is an elementary thing, it contains all the capacial gregaria of any oxygen. For if any other oxygen can be found with a less number of capacial gregaria, then the first jar was not elementary. But if we examine any elementary oxygen, as all other oxygen is a simile of that which we have examined, any experiment with certain oxygen giving a certain result will under like conditions give a simile of that result with any oxygen; and so also with any other primary aggregate existence. And the differential elements constitute the primary aggregate existences in which reside the capacial gregaria which are the primary causes of all effects in nature. These elements combine and form chemical compounds, which possess capacial gregaria different from those possessed by either of the elements entering into them.

Now every capacial gregarium possessed by a primary aggregate existence, is a sine qua non of that aggregation, and it has a simile of itself in every other aggregate existence, which is a simile of the given aggregation; and this is true also of all compounds. And hence we can experiment upon all aggregate existances, and by the homonical syllogism, infer from the result in any case the results in all cases of similical similia or differentia.

And the first things to be determined by observation or experiment, about aggregate existences, are the conditions of time and space by which their capacial gregaria are regulated. And if we find by observation or experiments upon nature that, a certain gregarium of a certain aggregation is conditioned in a certain manner, we know that a SIMILE of that gregarium in similar aggregations, will be conditioned in a similia manner, and in like manner and with like inferences, we may experiment with the gregaria in

fasciculo, with aggregate existences themselves. Having now cleared the way, as we hope, we will in the next chapter proceed to explain the conditions of time and space, which regulate the causal gregaria, and we will then see the manner of proceeding to some extent, and the reader will be able to understand better, what we mean by induction.

CHAPTER III.

CONDITIONS OF TIME AND SPACE.

We have already said that the conditions of causes and effects are time and space; we have also shown, that not an homonical gregarium but heteral gregaria are the causes of every effect. And in a previous chapter upon cause and effect we showed that, in every effect some homon becomes heteral or some heteral becomes homon, some similia become differentia or some differentia become similia, some commensura become incommensura or vice versa; this, we saw, is a condition of causation. And if we take two ivory balls, each of which possesses impenetrability, we must see that the impenetrabilities of the balls are heteral in space; but we must see also that, unless these heteral in space occupy an homonical time, i. e., unless their times be homon, no effect can be produced by them inter se. If an effect is to be produced at a certain point of time between two ivory balls, but before that point of time come, one of the balls be annihilated, we must see that the proposed effect can not transpire from the want of an homonical time for the two balls. Those existences, which existed yesterday but not to-day, can not be the causes of effects, which begin to transpire to-day, i. e., causes must possess an homonical time with that point in which the effect begins to transpire or originates. And hence let the effect be the removal of a cart from a certain place to another upon a hill, and let us take it for granted that some horse drew the cart up the hill, and suppose we wish to ascertain the individual horse that did it. In the first place we must ascertain the homonical time in which this effect occurred, then we may think of Bucephalus the horse of Alexander; but we know that he could not have done it, if the times of Bucephalus and of the effect are heteral. And we know that no other horse than one, whose time of existence is homonical with the time of the effect, could have done it. But any horse, whose time is homonical with that of the effect, may have done it, i. e., such horse fulfills the condition of time. And hence all aggregate existences possessing the causal gregaria of any given effect, must be synchronous with the transpiration of the effect, i. e., their times and the time of the beginning of the effect must be homon.

Let us next look into the conditions of space. For, as all the acting causes of any given effect must be synchronous and in space, we must determine the conditions of space; and where the conditions of space can be determined, we know that, no aggregate existence, outside of those conditions

in any given case, can be an aggregation, which contains the causal gregaria or a causal gregarium of the given effect. And we must remark that, where two aggregate existences contain the heterical gregaria, which are the causes of any given effect, these gregaria must operate through the space situated between the two aggregations.

A ————— B

If A and B be two aggregate existences with a certain space between them, and A put forth certain energies, these energies must take some direction in space; and unless they take the direction towards B, the energies of A and B can not meet in an homonical where, and unless the heterical energies come to an homonical where, no effect can follow.

D ————— A ————— B ————— C

Suppose, for instance, that the enegies of A take the direction only towards D, and the energies of B only towards C, then the spaces of these heterical energies will always remain hetera, and no effect can follow from these heterical energies inter se. The conditions of aggregate existences in space, therefore, necessary to causation, are that, the heterical gregaria possessing heterical wheres, shall find an homonical where, i. e., that their wheres shall in some point of space come to be homon. And it will readily be suggested that, though these energies may take the direction towards each other, yet they may not meet.

A — C — D — B

Thus: Suppose A to be a magnet and B an iron filing, if A's energies terminate at C and B's at D, then they have not found an homonical where, and therefore no effect can follow.

Now a homon of time and a homon of space are the conditions sine quibus non of causation; and all the gregaria, which can be causes, must come into these two homonical hetera. And hence whenever any effect takes place, these conditions have been fulfilled; and the object of inductive inquiry is to find, not only what objects fulfill these conditions, but also what objects operate, become acting causes in these conditions, i. e., what gregaria in these conditions are the sine quibus non of any given effect. And we must always recollect that, we must have a certain effect in view, and that effects inter se similia may be produced by sets of causes, inter se similia, i. e., by similical similia or similical differentia. And as gregaria are found in aggregations, we must first determine the aggregations containing the causal gregaria of any given effect to this task we will now proceed.

CHAPTER IV.

HETERICAL INDUCTION.

We have heretofore treated of simple heteration and shown that the power of the mind to heterate depends upon time and space. The succession

of our own thoughts in time enables us to heterate them, and the revolution of the earth and of the heavenly bodies in time enables us to fix upon any particular period of time and hold its relations in our mind. By the where of ourselves and the where of other objects in space and their relations inter se we are also enabled to locate a particular where in space and preserve its relations in our minds. And although we may not always be able to point out the precise point of time, in which a given effect begins to take place, we can generally come near enough to that period for the purposes of heterical induction; and so also we can come sufficiently near to the precise where in space of a given effect. Simple heteration is sufficient to bring us to the point of time and the point of space of any given effect under consideration of the inductive processes. And when we have the period of time in which any given effect took place, as the cause of that effect must have been inter se synchronous and have touched upon some homonical point of that period of time, no aggregations before or since that period could contain the causal gregaria of that effect. If the *Æneid* was written in the age of Augustus, no person, who lived and died before that age, or who has been born since, could have written it. And hence if we know the period of time in which any given effect took place, all aggregate existences, which have not an homonical time with that period, are immediately heterated from the causes of the effect by our minds; and this is heterical induction. For, when we have thrown out those existences, which could not have been the causes, we have before us other existences, which may have been the causes, and by casting out the former we have led in or inducted the latter. And were there but two aggregate existences in esse at the period of time of the effect, as there must have been heterical gregaria concerned in producing it, we would know by the heterical induction of aggregations by their times alone that, these two existences contained the causal gregaria of the effect.

But although the heteration of objects from the time in which any given effect takes place, by throwing out many aggregations which could not contain causes of the effect, narrow the field in which the causes are to be found. Yet there are afterwards so many aggregate existences in esse synchronous inter se and having times homonical with that of the effect, and any of which, therefore, so far as time is concerned, may have been causes of the effect, that after that we have determined the homonical time of the effect and determine also what existences have times homonical with this, we are still unable to tell which of these contemporary existences contained acting causes in the present instance. We have, therefore, to proceed farther and heterate the wheres of objects from the homonical where in which the effect took place. Although this be an easy matter in some instances, yet in others it is attended with great difficulties. If we see an object in motion by heterical impenetrabilities, if a ball be started by the impact

of some other object, every object, which at the homonical time of the effect's beginning, was outside of the homonical where of impact, i. e., whose where and the where of impact were hetera, can be immediately heterated by the mind from the causes of the effect. And so also, from the very nature of compounds, we know that, the ingredients compounded must come in contact or they would not enter into compounds together.

And although we can not tell but that other existences than those ingredients which enter into compounds, may have something to do with the compounding of those ingredients, yet if the action of these other existences be always constant at all times and places, whenever and wherever a given effect is offered to our senses, for all practical purposes their action may be omitted in our considerations without any error to our principles or results. Thus; although we may not be able to heterate the space, which bounds and limits the capacial gregaria of the north polar star, from the space in which pine shavings are burning, yet if the influence of the north star be constant whenever and wherever shavings and fire are found upon our earth, for all practical purposes we may omit this influence in our considerations and seek after other aggregations, whose capacial gregaria we can determine and limit in space; and if their space and the space in which shavings are burning be hetera, we may immediately heterate those other aggregations from the causes of the effect. And hence, whenever, for instance, we find soap, we feel assured that no ingredients outside of those which have come in contact, can contain the causes of soap, or at least we may look for and receive as causes, if not all of the causes, some capacial gregaria contained in the ingredients, which have come in contact when soap came into existence as an effect.

But in numerous instances, for the purposes of the heterical induction of aggregations in space, we must follow Bacon's rule of varying the circumstances, i. e., we must find what capacial gregaria of aggregate existences are within the homonical time and place of given effects in one and the other instance of similical effects. Sometimes by observation upon numerous instances of similical effects in nature, we are able to heterate aggregate existences from others containing the causal gregaria; and very frequently we can do this by experiment. If, in the consideration of compounds, for instance, a chemist can analyse and find a certain portion of water to contain the primary aggregations, oxygen, hydrogen and sulphur, in one instance, and in another instance, he find a portion of water to contain oxygen, hydrogen and potasium, he may then, by the latter instance, heterate sulphur from the sine quibus non of water; for, in the latter instance, water occurs without sulphur being in the homonical space of the effect: and by the former instance he can heterate potasium from the sine quibus non of the effect. But it is not quite clear from the above analysis of the chemist, that both potasium and sulphur can

be absent from the water; for, oxygen and hydrogen may not unite, for anything we yet know, into the compound of water, without the presence of either the one or the other of these substances. But if the chemist find a portion of water containing only oxygen and hydrogen, he may then heterate all other aggregations from the *sine quibus non* of water. But neither oxygen nor hydrogen can be heterated from the causes; for, they are, each of them, primary aggregations, and were one of them taken away, there would not be left heterical gregaria to produce an effect. Now if a chemist can take certain elements and by them produce a compound or any given result, the mode of making heterical inductions in the case is the same as in analysis. He must wait until he perceives the effect, before he can heterate any object from the causes of it. The only difference is that in analysis he must seek after the aggregations, which are in the homonical time and place of the effect in different instances, while in synthesis he already knows the aggregations in the homonical time and place of the effect without inquiring after them.

And in general, if we suppose any given effect, to contain in its homonical time and place, the aggregations represented by a, b, c and d, in one instance, and in another instance a, b, e, f, and in still another a, b, g, h, we may, from the consideration of these three instances, heterate each of the aggregations severally, excepting a and b, from the *sine quibus non* of the effect; though it is not certain that a and b alone could produce the effect without the presence of some of the others, unless we can find an instance in which they alone are present. Bacon's rule of varying the circumstances, or of examining different instances of similitudinal effects, it will be perceived, enables us to heterate, from the causes in certain cases, objects occupying the homonical time and space of an effect; one instance can be used to enable us to heterate some of the aggregations from the *sine quibus non* of another.

This matter of varying the circumstances and thereby gaining the data from which heterical induction can proceed may be explained in a little different manner from that already given, though it comes to the same thing. Thus; if we mix together three gasses represented respectively by a, b, and c, and we apply this mixture to a piece of white paper, for instance, and observe the change or effect, which takes place in the paper, and we then apply the three gasses, a, d and e, and observe also the effect upon the paper, and we find the two effects to be inter se similia, the latter instance enables us to heterate b and c from the *sine quibus non* of such similitudinal effects, and the former instance enables us to heterate d and e from the *sine quibus non*, leaving the effect to take place between the capacial gregaria of a and of the paper. If we represent the paper by x, we may say, a, b, c and x produce a given effect, which we observe upon x, but a similar effect is produced upon x by a, d, e and x, and therefore, b and c are not *sine quibus non* of such effects, nor are d and e. And if a, b and c, each of them, leave changes upon

the paper, which can be inter se discriminated, which changes may be represented respectively by the capitals A, B and C, and in an other instance, a, d and e, produce changes, which can be discriminated inter se, we may then find from the gregaria of a, b, c and x the effects a, b, c, and from the gregaria of a, d, e and x, the effects, a, d, e, and from these data we can heterate b and c, and d and e, from the sine quibus non of the effect a &c.

Heterical inductions are made daily in the transactions of life and always have been so made, though like the syllogistic process, the modus operandi of the mind has not been well understood. A very simple case of heterical induction is continually made before courts of law. If a man be indicted for murder and an alibi be proven, i. e., if it be clearly shown, that the person charged with the crime, was at the time when the crime was committed, a hundred miles from the place in which it was done, the accused is heterated from the causes of the murdered man's death. The principle of heterical induction may be summed up in the following heterical proposition; whatever is absent from the homonical time or place of a given effect, and the causes of that effect, are hetera.

CHAPTER V.

HOMONICAL INDUCTION.

In the previous chapter we explained the modus operandi of the mind in separating those aggregate existences, whose gregaria can not be causes of a given effect from other aggregations, whose gregaria may be the causes, so far as time and space are concerned, i. e., their times and wheres fulfill the conditions of causation. In the present chapter we must show the process of the mind in determining what aggregations fulfilling the conditions of time and space, and the aggregations containing the causal gregaria are homonical hetera. Although we may heterate all other objects from the homonical place of a given effect at the time the effect took place, excepting a, b, c, yet it is not certain that a, b, c, each of them, contain the causal gregaria of the given effect, nor is it certain which of them do contain causal gregaria. Three men may have hold of a rock when it begins to move, and yet one of them may have done all the lifting. And supposing that lye, sand, sawdust and adipose tissue be put together in a kettle and boiled, and soap be the result, which of these ingredients contained the causal gregaria of the effect? We might, no doubt, heterate some of these ingredients from the causes in the manner pointed out in the last chapter, but our object now is not to find existences, which in relation to the causes of the effect are heterical, but to find the aggregations, which are homonical with these containing the causes. And in order to find the homonical aggregations we must again follow Bacon's rule of varying the circumstances. Suppose we take lye, sawdust and sand without any adipose matter and boil them just as spoken of above, and

find that no soap is produced, we may then conclude that adipose matter was a *sine qua non* of soap in the first experiment. And hence when we wish to ascertain whether any one of the aggregations, fulfilling the conditions of the time and place of a given effect, be a *sine qua non* of that effect, we first ascertain, if possible, all the aggregations fulfilling those conditions, and then we find an other case having all the aggregations as before, excepting that aggregation, whose gregaria as *sine quibus non*, we wish to try; and if in the latter case the effect is not produced as in the former one, then this aggregation left out of the latter case was a *sine qua non* of the effect in the former case. Thus; if in one case we find the aggregations fulfilling the conditions of the time and place of the effect a, to be a, b, c, and d, and in an other case we find a, b and c without d in like conditions as before, without the effect a, we then have the data from which to make the homonical induction, that d was a *sine qua non* of a. That the sun is a *sine qua non* of day may be proven by taking the case of a bright day and a case in the same day, when the sun is eclipsed by the interposition of the opaque body of the moon, or when the earth revolves and takes us away from the sun.

And it is no matter which of the two cases, one of which contains all the aggregations and the other all excepting one, come under our observation first. If a, b, c and d, be found in certain conditions, and then e also come into those conditions and then the effect a immediately commence, all the data of the two cases required are furnished. Before the sun rises, we have the aggregations, a, b, c, . . . p without day; when the sun rises we have the aggregations a, b, c . . . p and the sun, and then it is day. And if we can find cases by which we can thus try successively each one of the aggregations fulfilling the conditions of time and space, we may find, by homonical induction, all of the aggregations containing all the causal gregaria of any given effect. But we must be sure that the case, in which the effect does not occur, contains all the aggregations excepting the one, which we are trying as to its being a *sine qua non*, and which, the case, in which the effect follows contains. Thus; if the case, in which the effect a, follows, contain the aggregations, a, b, c, d and e in an homonical time and place, and we wish to see whether a, was a *sine qua non* of that effect, we must find a case in which b, c, d and e are found in a similical time and place without the effect.

If there be more aggregations in the case in which the effect does not follow, i, e, if there be b, c, d, e and f in the case without the effect a, and a, b, c, d and e without f in the case where the effect follows, as the effect a does not follow in the former case, the additional aggregation f would not vitiate our inference respecting a's being a *sine qua non* in the latter case, unless some effect due to f should prevent the effect a in the former case. If a, b, c, d and e be found to make a compound in the condition g, and b, c, d, e and f

remain but a mixture in the condition *g*, we may infer *a* to have been a *sine qua non* in the former case, unless *f* be a preventing cause in the latter one. But for entire certainty it is necessary that the two cases agree in the aggregations except the one which we are trying. If we have a given effect *a*, with the aggregations *a*, *b*, *c* and *d*, in one case, and in an other case we have the aggregations *b* and *c* only and without the effect, we can not tell which or whether both *a* and *d* were not *sine quibus non* of the effect *a*, in the former case. The principle of homonical induction may be summed up in the following homonical proposition; whatever existences are *sine quibus non* in the homonical time and place of an effect and the causal gregaria of that effect, are homonical.

CHAPTER VI.

DIFFERENTIAL INDUCTION.

We have already seen that the homonical *a* and the homonical *a*, through their times are hetera, are in space homon, i. e., they are in the same where at any given point of time. We have also seen that the homonical *a* and the heterical *a*, though their times may be homon, are hetera in space, i. e., one *a*, has a certain where and the other *a*, has an other certain where, both of which wheres may be occupied at the same time. We have also seen that hetera lie at the foundation of causation, and that things inter se similia, and also things inter se differentia, must be inter se hetera; and hence either similia or differentia are the causes of every effect. The homonical *a*, and the heterical *a*, are inter se hetera, they are also inter se similia, but *a*, and *b*, are hetera and they are also inter se differentia.

Now as the gregaria of aggregate existences are the causes of all effects and as there must be heterical gregaria concerned in the production of every effect, and as the heterical gregaria concerned must be inter se similia or differentia, it is the province of differential induction to eliminate those gregaria, which, with reference to the causal gregaria of an effect existing in either of the aggregations in the homonical time and place of such effect, are differentia. And in order to do this, we must first make heterical and homonical inductions of aggregations, (we may then also make heterical inductions of gregaria, which is as far as Bacon pushed induction) and then we must make differential inductions in the method about to be explained. And in order to understand the matter thoroughly, let us approach the subject by first clearing the way. Suppose we take two aggregate existences, whose gregaria we know, and suppose the gregaria of the first aggregation to be, *a*, *b*, *c*, *d* and *e* and no more, and the gregaria of the second aggregation to be *a*, *b*, *g*, *h*, *i*, and no more, and suppose that in an homonical time and place, by heterical and homonical inductions of aggregations, a certain effect, which we will call *a*, to spring from these heterical aggregations; then we can not

tell, whether the effect a, sprung from the similia a and a, or b and b, or from the differentia a and b, b and b, or c and i &c. But supposing the effect to have sprung from but two heterical gregaria, these heterical gregaria must be located, one in each aggregation, and not both in the same aggregation, otherwise the effect would spring up in a single aggregation and the two aggregations would not be sine quibus non in the homonical time and place of such effect, as we may have determined to be the case by a previous homonical induction, and without a previous homonical induction of aggregation, differential induction of causal gregaria can not proceed.

But suppose we take five aggregations, whose gregaria we know, the the gregaria of the first being a, b, c, d and e, and no more; those of the second a, b, c, d, and f, and no more; those of the third a, b, c, e and f, and no more; those of the fourth a, b, d, e and f, and no more; those of the fifth a, c, d, e and f, and no more. Now we can conclude, by heterical induction, that the effect, which springs from the first and second aggregations, is not caused by the similia e and e, for e does not exist in the second aggregation; and the effect which springs from the first and third, is not caused by the similia d and d; and the effect, which springs from the first and fourth, is not caused by the similia c and c; and the effect, which springs from the first and fifth, is not caused by the similia b and b. If now the four effects be inter se similia and in view of the above state of the case, we look upon the second aggregation, we conclude by heterical induction that, in that aggregation e was not a sine qua non of the effect, which sprung from the combination of the first and second aggregations; and hence a simile of it is not a sine qua non in any other aggregation, which may combine with a simile of the first aggregation and produce a similical effect. And in the other instances, we may eliminate by heterical induction, d from the third aggregation, c from the fourth, and b from the fifth.

We have not been speaking above of any other effects than these arising from the given combinations of the given aggregations, which by previous heterical and homonical inductions we know to be the aggregations containing the causal gregaria, and the gregaria of each of which aggregations we know also. There may, for all that yet appears, however, be other aggregations containing causal gregaria of effects, which, with reference to the given effects spoken of above, are similia, and yet the causal gregaria of the other effects, with reference to the causal gregaria of the given effects, may be differentia. But suppose there be other aggregations containing other causal gregaria of an heterical effect A, these other causal gregaria, with reference to the causal gregaria of the homonical A, the effect above spoken of, must be either similical differentia, in which case the heterical effect is but another instance of like causes, i. e., the causal gregaria of the homonical A being the homonical differentia, a in the first

aggregation and *f* in the second, for instance, if the causal gregaria of an heterical *A'*, *A* and *A'* being inter se similia, be similical differentia, the causal gregaria of the heterical *A'* are the similical differentia *a'* and *f'*; or the causal gregaria of the heterical *A*, with reference to the causal gregaria of the homonical *A*, must be differential differentia, i. e., the causal gregaria of the homonical *A* being the homonical differentia *a* and *f*, for instance, the causal gregaria of a heterical *a*, may be the differential differentia *e* and *g*, for instance for aught that yet appears. But in no case, the causal gregaria of the homonical *A* being the homonical differentia *a* and *f*, can the causal gregaria of an heterical *A* be, with reference to the causal gregaria of the homonical *A*, similical similia; for the similia *a* and *a*, *b* and *b*, or *d* and *d*, &c., to be similical similia with the homonical differentia *a* and *b*, is absurd and impossible.

But supposing the causal gregaria of an homonical *A*, to be the differentia *a* and *f*, may not the causal gregaria of a similical *A*, be inter se similia, such as *k* and *k*, *y* and *y*, or *z* and *z*? Now if we contemplate the causal gregaria of the homonical *A*, and those of the similical *A*, as the two *a*'s are inter se similia in every respect, and as each of the causal gregaria of both *a*'s is not an aggregation but a simple gregarium, the effect produced by *a* and *f* inter se can not be a simile of an effect produced by *a* and *a*, inter se, so long as homon is homon, and similia are similia; and if *a* can originate upon *a'* a simile of the effect, which *f* originates upon *a*, then *a* and *f* must be inter se similia, which is absurd. If a certain vibration of the atmosphere in connection with the apparatus of the ear produce a certain sound, then a simile of that sound, the apparatus of the ear remaining the same, can not be produced but by a simile of the given vibration.

But in the case considered above, the causal gregaria in the first instance being by supposition the differentia *a* and *f*, and in the second instance the similia *a* and *a*, one of the causal gregaria (*a*) in the first instance and one (*a*) in the second are inter se similia; that no effects inter se similia can spring from such sets of causal gregaria, is evident. But an effect, an homonical *a*, having sprung from the causal gregaria, the homonical differentia *a* and *f*, may not a similical *A*, spring from the similia, *g* and *g*? In the first instance *a* originated upon *f*, an homonical effect *A*, and we see that *g* cannot originate upon *f*, a simile of *A*, unless *a* and *g* be inter se similia; but in the first instance, by changing the mode of expression without affecting in any manner, the result, *f* originated upon *a*, the homonical effect *A*, and *g* cannot originate upon *a*, a simile of *A*, unless *g* and *f* be inter se similia; but *a* is an homonical gregarium and *g* is an homonical gregarium, and inter se they are differentia. Now two gregaria inter se differentia can not in their action be inter se similia unless similia and differentia be inter se similia, which is impossible. And if *a* cannot act towards *f*, as *g* acts towards *g*, and if *f* cannot act towards *a*, as *g* acts towards *g*, the results of the actions between *a*

and f, and between g and g, can not be inter se similia. And an homonical effect a, having sprung from the homonical differentia a and f, we may reason in like manner respecting the effect, which must spring, if at all, from the differential differentia g and h. So too if an effect spring from the similia a and a, no similical effect can spring from the differentia a and b, c and d, &c., nor can a similical effect spring from differential similia as b and b, or c and c, &c. Of the differential elements of the alphabet, no other two can be conjoined so as to produce the sound resulting from ab; and so it must be throughout nature. And hence it must appear that effects inter se similia in every respect must be produced by similical gregaria, either similical similia or similical differentia; differential similia or differential differentia can not produce similical effects. And therefore if two or more aggregations come into the homonical time and place of an effect, we first find by heterical and homonical inductions of aggregations, the aggregations from which the effect sprung, then we look for other instances containing a simile of one of the aggregations from which a similical effect sprung, i. e., we vary the circumstances, and by doing so we are often able by heterical induction of gregaria to eliminate certain gregaria from the differential aggregations combined with the similia of the other aggregation in the given instance; then we proceed farther.

And it must be remembered that two gregarial similia cannot exist in the same aggregation. Thus; iron possesses hardness, and there is an homonical hardness in this piece and an heterical hardness in that piece, and inter se the homonical hardness and the heterical hardness are gregarial similia; but there cannot be two hardnesses in an homonical piece of iron; all the gregaria in a single piece or particle of iron are inter se differentia. Now when effects are produced between two aggregations, these aggregations either disappear in a measure and merge in the effects, as in chemical compounds, or the effects, which our senses witness are grounded in one of the aggregations or in both. When oxygen and hydrogen unite and form water, the two aggregations, in a measure merge in the effect—water, i. e., although the weight, impenetrability, &c., of the separate elements remain as gregaria of the compound, yet some of the gregaria of each element seem to have disappeared and to have merged in an effect, whose gregaria with reference to the gregaria of either of the elements are differentia; but if we apply oxygen to steel, we witness an effect grounded in the steel. Having now cleared the way, as we hope, we may proceed to differential induction.

Suppose then, that we take a certain aggregation, which we will call A, and that we apply the aggregation B to it, and we find a certain effect x to spring up; we then in like conditions, apply to A, or to a simile of A, the aggregation C, and find either no effect or the effect y, then it is certain, A and A

being homon or inter se similia in every respect, that the causal gregarium of x existing in b has no simile existing in C, i. e., that each of the gregaria of c and the causal gregarium of x existing in B are inter se differentia. Suppose then, that we can discover in B the gregaria a, b, c and d, for instance, and that we can also discover the gregaria a, b, c and d, in C, then we know that neither a simile of a, nor of b, nor of c, nor of d, was the causal gregarium, in B or in similia of B, of x, which sprung from the homonical time and place of A and B. And letting the capitals A, B, C, D, &c., be names to distinguish aggregations inter se, and the small letters, a, b, c, d, &c., be names to distinguish effects inter se, we may make the following tables to assist the understanding.

1st.	2d.
A and B produce a	B and A produce a
A and C produce b	B and C produce g
A and D produce c	B and D produce h
A and E produce d	B and E produce i
A and F produce e	B and F produce j
A and G produce f &c.	B and G produce k &c.

Now in the first set of instances in the homonical time and place of the effects, if we desire to find the causal gregaria of a, which exist in B, we see that gregaria, similical with the causal gregaria in B of the effect a, do not exist in C, nor D, nor E, &c., and hence wherever we find a gregarium in C, D, E &c., which has a simile in B, we know that this similical gregarium in B and the causal gregarium, or each of the causal gregaria in B, if there should be more than one causal gregarium in B, are inter se differentia. And in the second set of instances we may deal in like manner with the gregaria of A. And after that we have differentiated, by differential induction, as in the manner now explained above, the gregaria in B, which are not the causal gregaria, from the causal gregaria, we may dismiss the non causal gregaria from our consideration and look further into the matter.

The case, however, may and does occur in chemistry, where two aggregations will not produce an effect without a third aggregation being brought to bear upon them, and then differential induction is rendered still more complicated and difficult. Suppose that A, B and C, produce the effect a, and that A and B produce b, A and C produce c, and B and C produce d, then it is evident that the causal gregaria of a existing in A and each of the gregaria in B are differentia; for, if the causal gregaria of a in A, have similia in B, then B and C would produce a without A. And in like manner, it is evident that the causal gregaria of a in A and each of the gregaria in C are differentia, and the causal gregaria of a in B and each of the gregaria in A are differentia, and the causal gregaria of a in B and each of the gre

garia in C are differentia. And hence the proximate causal gregaria of a must be in b and C, or in c and B, or in d and A. Now if A and B really produce no effect at all, and if B and C produce no effect at all, it is evident that the proximate causal gregaria are in c and B. And if c be a permanent effect, we may then deal with c and with B in the manner above given; but if c be evanescent we are not able to manage it in that manner. If nitric acid and platinum in an homonical time and place produce no effect, and if silver and platinum in like conditions produce no effect, but nitric acid dissolve silver, i. e., nitric acid and silver produce an effect, which we will call c; and if nitric acid, silver and platinum produce an effect, which we will call a, then it is evident that the causal gregaria of a lie in c and platinum, and we must, if possible, inquire into the gregaria of c and also into those of platinum by differential induction as explained above.

But suppose, as before, that A, B and C produce the effect a, and that A and B actually produce b, and A and C produce c, and B and C produce d, it is then uncertain whether b and C, c and B, or d and A produce a; and if the effects, b, c, and d be evanescent and not of a permanent character per se, so that we cannot examine them, we can make no inductions respecting the proximate causal gregaria of a. If, however, b, c and d be of a permanent character, when A and B have produced b, we can try b with C, and so of c and d; and in this manner we can differentiate the gregaria of c and d from the causal gregaria of a.

When four elements enter into a compound in a binary manner, differential induction is easy. When A and B produce a, and C and D produce b, and if a and b be permanent effects and they produce c, we may first make differential inductions of the causal gregaria of a in A, and in B, of b in C and in D, and then of the causal gregaria of c in a and in b. But it may be that A, B, C and D contain the still more remote causal gregaria of a; A and B may produce b, A and C produce c, B and C produce f, or the operation may be still more complicated and then these resultant effects produce their effects and the last mentioned effects produce still others, and so on to a given effect x for instance. Organic and animal life is, no doubt, produced in this manner. But however complicated the matter may be, the principle of differential induction in any case has a simile in every other case, and it may be summed up in the following differential proposition; whatever gregaria being put in the conditions, in which certain causal gregaria produce a given effect, and they do not produce a simile of that effect and the causal gregaria of that effect are differentia.

CHAPTER VII.

SIMILICAL INDUCTION.

Having treated in the preceeding chapter of differential induction we will not find much difficulty in understanding similical induction, and we

need not spend much time upon the subject. But in order to assist the understanding let us represent aggregations by the capitals A, B, C, &c., and their effects by the small letters a, b, c &c., and let us form two tables as before:

1st.	2d.
A and B produce a	B and A produce a
A and C produce a	B and G produce a
A and D produce a, &c.	B and H produce a, &c.

Now in the first set of instances, as B, C and D, each of them along with A produce a, A remaining the same or a similitude of A being in each instance, the respect, in which B, C and D are inter se similia, is the causal gregarium of a, existing in B, in C and in D, &c.; and in the second set the respect in which A, G and H are inter se similia, is the causal gregarium of a existing in A. And if A and B produce d, and then d and C produce a, we may make a similitudinal induction respecting the causal gregarium in d and in C in the manner shown above. And if A and B produce d, and C and D produce g, and then d and g produce a, we may continue our inductions in like manner, and so on.

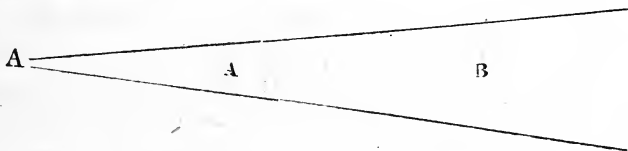
In differential induction the respect in which aggregations, one of which contains causal gregarium of a given effect and the others not, are inter se similia, and the causal gregarium in the one causal aggregation are inter se differentia; in similitudinal induction, the respect in which aggregations, all of which contain causal gregarium of a given effect, are inter se similia, and the causal gregarium of the given effect in any one of the aggregations compared are inter se similia. And if two aggregations containing causal gregarium of a given effect and compared in the manner above stated be inter se similia only in one respect, that respect is the causal gregarium of the effect existing in each of the aggregations. The principle of similitudinal induction may be summed up in the following similitudinal proposition; whatever gregarium in similitudinal conditions produce similitudinal effects, are inter se similia.

CHAPTER VIII.

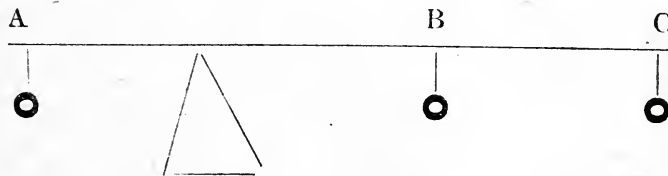
INCOMMENSURAL INDUCTION.

We have seen heretofore that, commensura and incommensura are relations which have an homological standard, and therefore when these terms are applied to aggregate existences, or to gregarium, they are applicable only to those existences, which are inter se similia. Thus: a may be equal to a', i. e., $a=a'$, or $a < a'$, a and a' being inter se similia; but if a and b be inter se differentia, a cannot be equal to b, i. e., $a=b$, and $a < b$, are propositions without any meaning, just as much as when we say that this sound is equal to that color. Now all incommensural effects are inter se incommensura, by reason of the incommensural relations of time or of space or of both, existing between the aggregation in which the effect is grounded and the other aggregation containing causal gregarium; or else by reason of the incommensural

relations between the quantities of the causal gregaria at homonical or heterical times, the times of application remaining commensura, and the spaces between the aggregations remaining commensura. Thus; an hour and a day are incommensural relations of time, and a steady rain for one hour and a steady rain for one day leave incommensural effects grounded in the land from the incommensural relations of their times, the quantities of rain falling in commensural times being commensura. And it is evident that, in heterical instants of time the causes of the effects grounded in the land, the rain which falls, are not homonical but similical; yet commensural quantities falling in commensural times, the effects will be incommensura from the incommensural relations of the times of the similical causal gregaria in operation to produce the sums total of the effects. Again; in the radiation of influences, the effects of those influences will be inter se incommensura from incommensural relations of space, the times, and quantities of gregaria in aggregations, in which the effects are grounded, being inter se commensura. Thus:



If A be a body radiating heat, for instance, a body at A will receive, in commensural times, more of the radiated influence than a similar and commensural body at B, i. e., the effect grounded in the body at A and the effect grounded in the body at B will be incommensura. Again:



If there be two pieces of iron, whose weights are inter se commensura, attached to the lever A C, the one at B and the other at C, their forces exerted upon a body at A will be inter se incommensura from their incommensural relations of space from the fulcrum. And again; if in commensural relation of time and space, incommensural quantities exert their influences, the effects will be incommensura.

And we must bear in mind that the incommensural effects, which are to be the subjects of incommensural induction, are grounded, and witnessed by our senses, in one of the aggregations containing causal gregaria, and our object is to find the other aggregation or aggregations containing the remaining causal gregaria. Thus; if A and B at one time produce a, and at a

other time A and B produce a' , and $a < a'$, these incommensural effects are grounded and witnessed by our senses, either in A or in B. When oxygen supports the combustion of coal, the effect, which our senses witness, is grounded in the coal. Now it is evident that, if A and B produce no effect whatever upon each other, they cannot produce incommensural effects: If A or B incommensurate a, A or B must be an aggregation containing causal gregaria. And hence, letting B be the name of simillical aggregations, if we find the simillical effects, named a, grounded in B, and these effects be inter se incommensura, we may then look for some other aggregation, A for instance, and make observations or try experiments with A and B, and by incommensural induction determine whether or not A contain causal gregaria of a. Thus; commencing with incommensural relations of space, suppose the effect a grounded in B to be incommensurated when B approaches or recedes from A; if now the position of A in relation to other aggregations be changed, i. e., if the other aggregations among which A is situated, be heterated by changing the position of A or of B, and a be incommensurated when B approaches or recedes from A, then it is evident that A contains causal gregaria of a. That the earth contains causal gregaria of the gravitation of terrestrial bodies towards its center is evident by incommensural induction. On the opposite sides of the earth at the same time, when the same stars contain between them and the earth one set of terrestrial bodies and the earth is between those stars and an other set of terrestrial bodies, the gravities or effects grounded in both sets of bodies become incommensura at incommensural distances from the earth's surface.

In the foregoing example we have seen that, from the incommensural relations of space between aggregations containing causal gregaria incommensural effects arise. Incommensural quantities or intensities of causal gregaria, their times and spaces remaining commensura, produce also incommensural effects. If a barometer be placed under the receiver of an air pump, and the quantity of air be increased and again diminished, and such incommensural quantities be attended with incommensural effects upon the barometer and the influence of all other objects be heterated from the homonical time and place of the effects, it is evident that the pressure of the atmosphere is the cause of such effects. And hence when effects grounded in certain aggregations are incommensurated and we can perceive by observation, and still more when we can make the experiment, that the quantities or intensities of gregaria in some other aggregation are correlative incommensura, and we can also heterate other objects, we may be assured that the correlative incommensura are connected with the effects by causation.

And again: the relations of time may enable us to make incommensural induction of the causes of incommensural effects. Although we can

not in one day or in one year perceive any material change in the falls of Niagara, yet other objects being heterated, and the water continuing to flow over from year to year and very gradual changes continuing to take place and being incommensura in incommensural times, other things being equal, from the incommensural relations of the times of the flowing and of the wearing away of the rock, we can infer the water to contain causal gregaria, in the absence of other experience.

The principle of incommensural induction may be stated in the following incommensural propositions: The relations of the times of causal gregaria to incommensural effects, spaces and quantities being commensura, are incommensura; the relations of the spaces of causal gregaria to incommensural effects, times and quantities being commensura, are incommensura; and the relations of quantities of causal gregaria to incommensural effects, times and spaces being commensura, are incommensura.

CHAPTER IX.

COMMENSURAL INDUCTION.

We have seen in the previous chapter that, incommensural effects, times being commensura, depend upon incommensural relations of spaces or of quantities between the causal gregaria; and, on the other hand, commensural effects, whose times are commensura, depend upon commensural relations of quantities or of spaces between the causal gregaria. And we must always bear in mind that, not an homonical gregarium but heterical gregaria are the causes of all effects, and that some of the causal gregaria are contained in the aggregation in which our senses witness the effect grounded, and some of them in some other aggregation, for which we are seeking as the cause of the phenomenon. When a magnet attracts iron-filings, some of the causal gregaria are in the magnet and others in the filings. And it is quite evident that, if we represent the quantity of the causal gregaria existing in a certain magnet by A, and the quantity existing in a certain piece of iron by B, and the iron of the weight C be attracted through a certain space A in the time D, a magnet containing 2A will attract iron containing 2B and of the weight 2C through the space A in the time D. If twelve pounds weight attached by a cord will raise twenty pounds upon an inclined plane through the space A in the time D, twenty-four pounds in like manner will raise forty pounds. And as in incommensural so in commensural induction, we must look to the relations of the effects, which we witness, and then to the relations of times spaces and quantities of other aggregations to these effects. And we have already remarked that, neither incommensural nor commensural induction has any reference to kinds of effects, but that on the contrary the effects, whether they be inter se commensura or incommensura, are always inter se similia.

Suppose then that by observation or experiment we find, first by an

homonical induction, A and B to produce A in the space b in the time c, and in other parts of space we find a simile of A grounded in a simile of B; we must then look to A or for some simile of A, in the respect of the causal gregaria of A existing in A; and in order to determine which object is this simile of A, we must examine the quantity of causal gregaria in A and in B, and their relations in B, and also the quantity of gregaria in the simile of B, in which we witness the effect, and the relations of this simile of B in space with other objects. And if we find an object, which we may call y, whose relation to the simile of B in space is commensural with the relation of A to B, times and the effects, A and a' being commensura, so far y is indicated as containing causal gregaria of a'. And if now by a change of spaces we can heterate other objects from relations similical with the relations of A to B, we can then fairly conclude that y, contains causal gregaria of the effect a'. The principle of commensural induction may be summed up in the following commensural propositions: The relations of space between the causal gregaria of commensural effects, times and quantities being commensura, are inter se commensura; the relations of quantities of the causal gregaria of commensural effects, times and spaces being commensura, are inter se commensura; and the relations of times of the causal gregaria, of commensural effects, quantities and spaces being commensura, are inter se commensura.

CHAPTER X.

INDUCTION PROMISCUOUSLY.

From what has been said in the previous chapters in this book, it must appear that in making inductions we use for the most part two cases at least, in which the aggregations are not homonical hetera, but homonical and heterical hetera. Thus: if we wish to make an heterical induction of the aggregations A, B and C, which in one instance we find to be in the homonical time and place from which spring the effect Z, we look for another instance of the effect Z, in whose time and place A nor a simile of A is not present: and in this latter instance, we do not find the homonical B and C, but we find similical B and C. And hence all induction proceeds upon the truth, that the laws of nature are uniform, or that similical or commensural causes in like conditions always produce similical or commensural results: and this truth, as we have seen heretofore, is established in our minds by the homonical syllogism. And in order to make even heterical inductions, we must have experience gained by observation or experiment, and this experience depends upon the powers of the mind to recognize homon, hetera, similia, differentia, commensura and incommensura. We find by experience, for instance, that a certain piece of soap will not cleanse any object, with which it does not come in contact; and if now we call this certain piece A, by the homonical syllogism, a simile of A will be conditioned in a similar manner: and hence if we find

an instance of cleansing in whose place a simile of A was not present, we make the heterical induction that A is not the cause of cleansing in this instance, and not a *sine qua non* of such effects. Heterical induction of aggregations, indeed, goes no farther than the particular instance from which a certain aggregation has been heterated. If, for instance, A, B and C are the only aggregations present when the effect Z comes into existence, and supposing A to have been a cause of Z, in this particular instance, we know by heterical induction, that R was not a cause of the homonical Z, but for all that we do not know that R, if in the place of A, would not be a cause of a similical Z. For the causal gregaria existing in A may have similical gregaria existing in R, and hence R would also be a cause of such effects as Z. Heterical induction of aggregations does no more than remove from an instance of a certain effect, certain aggregations as *sine quibus non*, and thus clear the way for further investigation.

Homonical induction proves directly causation in the instance to which it is applied; but the homonical induction of aggregations, although it prove a certain aggregation to be a *sine qua non* of a particular effect, yet it does not prove similical aggregations to be *sine quibus non* of effects similical with that particular one. Thus, if we find the aggregations, A, B and C in the time and place from which spring the effect Z, and by observation or experiment we find B and C without A in a similical time and place, and no effect follows, we can conclude that A was a *sine qua non* of that homonical Z, but we can not conclude that similia of A are *sine quibus non* of similia of Z. For although A and D as aggregations may be differentia, yet the causal gregaria of homonical Z existing in A may have similical gregaria in D; and hence D also will contain causal gregaria of similia of Z. Arsenic, copper and lead, as aggregations, are *inter se* differentia, yet in some respects they all contain similical gregaria, and hence each of them is a poison. And though by homonical induction of aggregations we prove a cause of similical effects, yet we do not prove the only cause. But if we can make an homonical induction of gregaria, we will prove the only causes of similical effects. It is very seldom, however, that we are able to obtain the data, either by observation or experiment, from which we can make an homonical induction of gregaria, and in order to make inductions of gregaria we are obliged to resort to differential and similical inductions.

In differential induction, which presupposes homonical induction of aggregations, we look directly at the gregaria of aggregations, and having applied these aggregations severally to a common substance, or to substances entirely similia *inter se*, we note the gregaria, which are *inter se* similia in two substances, one of which along with the substance A for instance, will produce the effect Z, while the other along with the substance A will not produce a simile of Z, and then we differentiate those similia from the causal

gregaria. Sugar and soda, for instance, will both dissolve in pure water, these capacial gregaria of the two substances are inter se similia; but when vinegar is applied to soda it will foam and boil, while when applied to sugar it will not; the capacial gregarium of being held in solution, therefore, is not in soda the cause of the ebullition witnessed when it is put into acid. In the first book of this volume we spoke of facial and capacial gregaria; we called the color, the taste, the feeling, the smell and the sound of objects, their facial gregaria, because they present such appearances to our senses. In reality, however, all these things are capacial gregaria; and the only difference is, that facial gregaria are perceptual facts immediately noticed by the mind, while our knowledge of what we have called capacial gregaria is derived from a comparison of perceptual facts. Thus, if I apply sugar to my tongue an effect is produced immediately between the sugar and my organs of taste; but if I put a lump of sugar in water, I see the sugar and the water and I may see the sugar dissolving; I, indeed, make an induction in every instance to arrive at the knowledge of capacial gregaria of aggregations. Now in making differential inductions, we always arrive at the knowledge of similical gregaria in various substances by observing the facts which spring from them when applied to similical substances. Thus, supposing our organs of taste to remain in similical conditions during a certain time, and during this time we taste two substances and find their tastes to be exactly alike: if now we find the one when taken into the stomach will act as an emetic and the other as a cathartic, we feel assured that the qualities, the gregaria which are similia in regard to our taste, and the gregaria, which produce in the stomach differential effects, must be inter se differentia. And so we may try any two or more substances with pure water or with any other thing, and in this manner determine similical gregarial, and if then we apply these substances to some other thing and find differential effects, we may differentiate the similical gregaria from the causal gregaria of a given effect. Differential induction does not, indeed, determine what gregaria are causal gregaria, but it merely determines what gregaria are not causal gregaria. And this it does not only in respect to a particular instance but in respect to all instances of similical effects. In the complicated workings of nature, however, laws are frequently antagonistic, and when one prevails over another, the prevailing one must always be considered the cause of the ensuing change which takes place, while the abrogated law, as it were, is not the cause although it is often called so. And in order to make the subjects of differential and similical induction clear, it is necessary to speak of this matter here. If, for instance, two men with rope and pullies be raising a rock and the rope break and the rock fall to the ground, we are apt to say that the breaking of the rope is the cause of the rock's falling, while in truth the causal gregaria of the rock's falling are in

the earth and rock, and the rope has nothing to do with it; though the rope, before it broke, was a cause of the rocks rising. Every change, indeed, is an effect, and when a certain positive phenomenon is going on it is being or has been produced by certain causes, some of which may cease to act and then the phenomenon disappears, in which case we are accustomed to call the CESSATION of the cause of its production, the cause of its disappearance. We are accustomed to say that the WANT of water is the cause of the death of a fish up on the land. That, however, which is heterated, the absence of a thing the want of an aggregation or gregarium, can not be the cause of anything. Certain laws may be kept in operation by certain gregaria of aggregations, and then certain phenomena exist; take away one of the aggregations, the taking away of which is truly an effect, and although we may properly call this taking away of the aggregation the REASON of the cessation of the phenomenon, yet it is not the cause of such cessation. That only which acts can be a cause. And hence although there may be and is plurality of causes of similical effects, i. e., the causes of similical effects are hetera, yet similical effects can not be produced by differential causes. And hence, although many aggregations, which as aggregations are inter se differentia, may produce similical effects, yet when we come to the causal gregaria of similical effects, the causal gregaria will always be similical. And therefore, the causal gregaria of similical effects being inter se similical, we at once know that, of two aggregations, one of which produces the effect and the other not, the gregaria which are inter se similia and the causal gregaria are inter se differentia.

In similical induction we compare together different aggregations, each of which we find to contain causal gregaria of similical effects to ascertain in what they agree. And if they agree but in one respect, this respect we know must be a causal gregarium: for the causes of similical effects are inter se similia. If they agree in several respects, we can not tell which of the similia are causal gregaria, and we should try by differential induction to differentiate some of these similia from the causal gregaria. Thus: if A, B, C and D will, each of them, with G produce similical effects, and if they all agree in several respects so that we can not tell the causal gregarium in either of them, we may find an aggregation in which some of the gregaria existing as similia in A, B, C and D, exist also, and yet the aggregation along with G will not produce the effect. That crystalline structure is not the causal gregarium of the double refraction of light is clearly proven by differential induction, although all substances which have hitherto been found to cause the double refraction of light, have been crystalline; and therefore, if we knew that they did not agree in any other respect, by similical induction, it would be proven, that double refraction depended upon crystalline structure alone. Crystalline structure may, indeed, be one of the causal gregaria exist-

ing in all substances, which refract light in this manner; but it is either not a cause at all, or at best it is not of itself THE cause, since all crystalline substances do not cause double refraction. Differential and similitudinal inductions aid each other in the search after causes, and neither of them should be neglected in any case, if they can be applied.

Incommensural and commensural inductions also aid each other in science. That the oscillations of the pendulum are caused by the earth, i. e., that the earth contains causal gregaria of these oscillations, and also that the earth contains causal gregaria of the gravity of terrestrial objects, was proven by incommensural induction; and then Newton by commensural induction proved the earth to contain also causal gregaria of the motion of the moon, and established what is called the universal law of gravitation. It does not seem to me to be necessary to speak farther upon the six methods of making inductions which we have endeavored to exhibit in the previous pages. These six methods of induction with the aid of ratiocination exhaust the powers of the human mind in drawing logical conclusions. And while treating of our subject in the first book, we saw that heteral lie at the foundations of knowledge and that homon is at the foundation of propositions; and we must now see that homon is at the foundation of all induction and that the homonical syllogism, sustains the truths upon which every induction proceeds.

But before passing on to further considerations it seems necessary to make a few remarks upon the methods of induction which have been set out by J. Stuart Mill, and in doing so we will not go into a lengthy discussion, as we believe that the student who has mastered the preceding pages of this book, will be able with but few suggestions, to perceive, what we consider, the errors of Mr. Mill. Of Mr. Mill's method of Residues, we shall merely remark that when we have subducted from any phenomena, what by previous inductions and ratiocinations we already know to be due to known causes, we proceed with the residue by some one or other of the six methods, which we have given, and that there is nothing peculiar to his method of residues, so that it should be considered in itself a particular kind of induction.

In what Mr. Mill calls the method of agreement there is the mixing together and confounding of what we have called heterical induction with similitudinal induction. The axiom upon which Mr. Mill considers this method to rest, to-wit: "Whatever circumstances can be excluded, without prejudice to the phenomenon, or can be absent notwithstanding its presence, is not connected with it in the way of causation," is applicable only to heterical induction, yet Mr. Mill endeavors to apply his method of agreement to infer causation from the agreement in respect to the presence of some antecedent in

every case from which the effect arises, which can be done only by similitudinal induction.

Mr. Mill's method of Difference corresponds with what we have called homological inductions, though his exposition of it has not been satisfactory to our mind. What Mr. Mill calls the Joint Method of Agreement and Difference, we regard as an intermixture of homological induction with erroneous views, which indeed, have reference to differential induction, although Mr. Mill had no conception of such method. It is, indeed, quite evident, that if A will produce a certain effect and B will not, the causal gregaria existing in A have no similia existing in B, and if now we could examine every substance which will not produce the given effect and find that they all agree in not containing some gregarium which is contained by A, there would be a strong probability, and nothing more than a probability, that this gregarium was a cause of the given effect. To pursue such a method, however, would be to depart from true induction and in the labyrinths of nature it is entirely impractical, and of very little value could it be done. On the other hand if we have but two cases, in one of which the effect springs from A, B and C, while in the other, viz: A and B, the effect will not be produced, although we may never be able by experiment to remove and again replace C, yet the two cases furnish all the data necessary for making the homological induction that C contains causal gregaria of the effect. We conclude that there is nothing in Mr. Mill's Joint Method to make it a particular kind of induction and further that a great part of his doctrine respecting it is erroneous.

Of Mr. Mill's method of concomitant variations, we will only say that he does not make any reference to what we consider to be the true principles involved in the matter, but treats of cases, some of which are to be determined by commensural and others by incommensural induction.

We have been very limited in our remarks upon the methods of Mr. Mill, as we desire in this book to take the affirmative and not the negative side of questions. Our object is to build up and not to tear down. And we propose also to make this book as concise as possible and not fill and enlarge it with criticisms. We may dismiss the subject of the inductive methods here, hoping that the reader will be able to understand the matter.

CHAPTER XI.

HYPOTHESES.

In the previous pages, we have dealt only with those principles which are brought into view by the comparisons of truths which have been derived from actual facts. And in the investigation of nature, our object must always be to find out what actually exists and how it operates, and not to assume certain hypotheses and from them determine how nature should exist and

operate. He, who would gain any scientific knowledge of the phenomena of nature, must investigate and not make assumptions. When we have really gained any new truth in nature, we do not rest the evidence of that truth upon an hypothesis; but in regard to all certain knowledge, we apply the saying of Newton "Hypotheses non fingo." Yet it is natural for man to form theories, and these theories often direct his energies towards valuable results. And for the purpose of stimulating the mind to investigation an hypothesis may be laid down, and in many instances for that purpose an hypothesis must be resorted to. No man, whose object is to search after truth, will take the trouble of investigating anything unless he expects to find out whether something which he has in view be true or not. A scientific hypothesis, therefore, is a subject stated for debate, in which arguments pro and con can be brought from actual facts in nature. If by ratiocination and induction founded upon actual phenomena, the hypothesis can be proven, that closes the debate and the hypothesis is converted into a truth, the evidence of which does not at all rest upon the hypothesis. And hence when we have laid down an hypothesis, our object must be to prove or disprove it from actual phenomena. But from nature we can prove only homon or homa, hetera, similia, differentia, commensura and incommensura; and therefore, scientific hypotheses may be divided into homonical, hetera, similical, differential, commensural and incommensural hypotheses.

In heterical hypotheses, which seem to be the most convenient to be treated of first in order, we may make a supposition respecting the heterical existences of a phenomenon; or granting its homonical existence, we may lay down an heterical hypothesis respecting its causal gregaria as sine quibus non of certain effects. Thus: as a simple example of a supposition respecting the heterical existence of a phenomenon; suppose we see a certain horse in an enclosure to-day, and to-morrow we see a horse in another place so much like the former that we are uncertain whether it be the same horse which we first saw, we may make the heterical hypothesis that, it was not the same one, i. e., this horse and the one we first saw are hetera, and then we must look for the evidence to prove the hypothesis. And if by investigation we find that the first horse has been continuously and is now in the same enclosure, we have proven the hypothesis to be a truth, whose evidence does not rest upon an hypothesis, but upon actual relations of time and space. And a similar example might be given to illustrate homonical hypotheses respecting the homonical existence of a phenomenon: we need not, therefore, speak of this again under the head of homonical hypotheses. But suppositions respecting the causes of phenomena are also useful to excite endeavors, and we may make heterical hypotheses respecting causation. If the aggregations A, B, C and D be in the homonical time and place from which springs the effect R, we may suppose, for instance, that D is not a sine qua non of the

R; and to prove our hypothesis we find another instance of the effect R, from which D was absent in time or space. And again: respecting causal gregaria, if the aggregation A along with Z will produce a given effect, and B also along with Z will produce a similar effect, and we can perceive that A possesses gregaria, which B does not, we may heterate those gregaria contained by A, but not by B, from the causal gregaria of the effect produced by A and Z, and thus prove the heterical hypothesis respecting those gregaria, if we have made one. And we have already, no doubt, gone far enough to see that heterical hypotheses, respecting the existence of any phenomenon, to be worth anything, must be susceptible of proof by simple heteration, and that heterical hypotheses respecting causation must be proven by heterical induction.

Homonical hypotheses also respecting causation must be proven by homonical induction; and until they are so proven, they are not, of course, to be received as really true, however useful they may be in stimulating inquiry. Homonical inductions, indeed, are best and more frequently made by experiments than by observations upon nature in her undisturbed processes offered gratuitously to our senses, and therefore we would more frequently resort to experiments to prove any homonical hypothesis. If, for instance, we should suppose that, it is the equal pressure of the atmosphere upon unequally balanced columns of water, which force the water up the shorter arm of a syphon, we could make experiments from which an homonical induction of the real cause could be brought out and the hypothesis proven. That there is an ether pervading all space and causing light by its vibrations, however, can not be proven by homonical induction, and if ever proven, (and without being proven the hypothesis amounts to nothing) it must be proven by similitical induction. An homonical induction can not be made in any case, unless the existence of aggregations containing causal gregaria can first be proven. If, for instance, we suppose that the aggregations A, B and C, produce x, when we do not know, whether or not, A really has an existence, we can make no homonical induction in the case; for although we should find that B and C alone will not produce x, that is no evidence of the agency or existence of A in the former case. Homonical hypotheses respecting causation, to be useful in increasing our stock of knowledge, must be susceptible of proof by homonical induction. And no hypotheses respecting the existence of an aggregation containing causal gregaria can be thus proven.

We may also make differential hypotheses respecting causal gregaria, and for their proof we must resort to differential induction. We might suppose, for instance, that the quality of dissolving upon the tongue and the causal gregaria of the taste in common salt are differentia; and by examining other substances containing this quality, we could prove our hypothesis. And in the examination of nature, as differential inductions, though they do

not prove what the causal gregaria are, assist very much in making similital inductions, so differential hypotheses should be assumed and tried that we may have every help in unravelling nature's complications.

In similital hypotheses we assume that, the causal gregaria of certain phenomena, whose causes we wish to ascertain, and the gregaria of certain objects, with which we are familiar, are similia: and if their effects can be shown to be inter se similia, we prove the hypothesis. Thus; if we find a particular color upon white paper, we may assume that the aggregation whatever it might have been, containing causal gregaria of such effect, was similar, in respect to its causal gregaria, to some object with which we are, familiar; and if the object with which we are familiar will produce upon the same kind of paper the same kind of color, we prove the hypothesis. If all the planets contain the quality of attracting iron, they, each of them, possess gregaria similar to the lode stone. And if we could make ourselves certain of the existence in any place, of an ether, whose vibrations would produce light, we could prove the ethereal hypothesis.

Respecting incommensural effects, we may make three suppositions, viz: first, that the times and spaces being commensura, the increase of the quantity of gregaria in a certain object incommensurates the effects; second, that times and quantities being commensura, the incommensural effects depend upon incommensural relations of space; and third, that spaces and quantities being commensura, the incommensural effects depend upon incommensural relations of time. And having made our hypothesis, we must then find the proof by looking into circumstances varied in these respects, and in which the effects occurs. But in making our hypotheses, these hypotheses must have reference only to what object or objects contain causal gregaria of the incommensural effects, which we witness. And we have remarked several times already that, in the cases from which an incommensural induction can be made, we are to deal only with similia, commensura and incommensura being relations inter similia. And the hypotheses above spoken of must be proven by incommensural induction. After having ascertained that certain objects contain causal gregaria of given effects, we may make hypotheses respecting the relative increase or decrease of the effects to the times, spaces or quantities of causal gregaria. But these hypotheses can not be verified by induction, and unless they can be verified by mathematical calculations, they are merely guesses. We are frequently obliged to make mathematical calculations respecting the laws of variation in the effects depending upon incommensural spaces and times. That gravity varies inversely as the square of the distance is not an induction, but a truth found out by the application of mathematics to actual phenomena. That the spaces passed over in successive commensural times by falling bodies are in the relation of the odd numbers 1, 3, 5, 7, &c., is a truth of the same kind, i. e., it is found

by making calculations of what actually occurs, as observed, in this respect, when bodies fall without being impeded.

Respecting commensural effects, we may make hypotheses in the same manner as respecting incommensural effects, and we must seek for the proof in like manner. We do not consider it necessary to make further remarks upon hypotheses. Every hypothesis respecting causation must be proved by induction; hypotheses respecting the relations of quantities, times and spaces are to be dealt with by ratiocination.

We have now completed our view of ratiocination and induction, so far as we propose to treat of them in common language. And we may well consider of what value these speculations may be to the cause of science. And merely as a speculation we regard the previous pages as not entirely unworthy of study; but we hope yet to show, that practical results of the grandest kind may be expected to follow from a knowledge of the principles therein set forth. To gather up an exhibit these principles in formulæ, and to apply them to the actual phenomenon of nature will be our object in Book III.

BOOK III.

CHAPTER I.

SIGNS IN RATIOCINATION.

In the two previous books we have examined the foundations of reasoning throughout and have endeavored to explain, by the use of common language, what we have considered necessary on the subjects of ratiocination and induction. Common language, however, is not the appropriate vehicle of recondite science. Without the assistance of symbols, which form a peculiar language, Algebra, which consists of syllogisms with commensural and incommensural propositions, could not have been brought to any great perfection. These commensural and incommensural propositions, with the syllogisms constructed upon them, however, have been expressed and wrought into Algebraic formulæ, which can be transformed in various ways, and thereby unexpected and grand results can be brought to our apprehension. And it may be useful to inquire whether the other four kinds of propositions also can not be expressed in symbols and reduced to formulæ, which may be formed into a complete system of abstract and exact science. That such complete system of science may and will be constructed in the future by the genius of man, the author of this treatise believes; and it seems to him to be not an unworthy undertaking to make a beginning at its construction, which may be an incentive to call to the work others of more favored circumstances and greater learning. The construction of such system will, therefore, be attempted in this book. And we will commence with simple propositions.

Let the sign \wedge stand for an homonical comparison; then, $a \wedge a$, will be equivalent to the proposition in common language, a and a are homon. Let the sign \vee stand for an heterical comparison; then $a \vee a$ will be equivalent to the proposition in common language, a and a are hetera. Let the sign \parallel stand for a similical comparison; then $a \parallel a$, will be equivalent to the proposition in common language, a and a are similia. Let the sign \div stand for a differential comparison; then $a \div b$ will be equivalent to a and b are differentia. Let the sign $=$ stand (as in Algebra) for a commensural comparison; then, $a = a$ will mean that a and a are commensura. Let the signs $>$ and $<$ stand (as in Algebra) for an incommensural comparison: then, $a > a$, or $a < a$, will mean that a and a are incommensura.

Now by the use of the foregoing signs, we can combine the six kinds of propositions in all the figures and modes of the syllogism. Thus in mode 1st:

$$\begin{array}{c} a \wedge a \text{ or } a \vee a \\ || \text{ or } \vdash \\ a' \wedge a' \text{ or } a' \wedge b \\ \therefore a || a' \text{ or } \therefore a \vdash a' \end{array}$$

And these syllogisms will be true irrespective of time and space, i.e., if $a \wedge a$ or if $a \vee a$, or if $a || a$, &c. to-day, they always have been and always will be in a like comparison, so far as time and space, as agents, are concerned.

But before proceeding further, it is necessary to explain the manner in which simple gregaria of aggregations by the use of signs. Let the first large letters of Alphabet, A, B, C, &c., stand for aggregations, and the first small letters, a, b, c, &c., for gregaria, then a syllogism in mode 1st may be thus constructed:

$$\begin{array}{c} a \text{ of } A \wedge b \text{ or, } a \text{ of } A \vee b \\ || \text{ or, } \vdash \\ a \text{ of } B \wedge b \text{ or, } a \text{ of } B \vee c \\ \therefore a \text{ of } A || a \text{ of } B. \text{ or, } \therefore a \text{ of } A \vdash a \text{ of } B. \end{array}$$

Now if a stand for the gregarium—color, we may interpret the syllogism thus:

$$\begin{array}{c} \text{Color of } A \wedge b \text{ or, Color of } A \vee b \\ || \text{ or, } \vdash \\ \text{Color of } B \wedge b \text{ or, Color of } B \vee c \\ \therefore \text{Color of } A || \text{color of } B. \text{ or, } \therefore \text{Color of } A \vdash \text{color of } B \end{array}$$

And these signs as above given are sufficient for all the purposes of the singular syllogism and of the singular homonical syllogism.

But for the purposes of the Plural syllogism, we wish signs, not only to express the comparison between the terms of the propositions but to show also the comparisons between the existences exhibited in each term. And for this purpose, we need but combine the signs already given, and reading from the left to right, interpret the sign on the left hand as an adjective and the succeeding sign as a noun. The following table will show the use of the signs:

Let the sign, $\wedge \wedge$, indicate homonical homa.

"	"	"	$\wedge \vee$	"	"	hetera.
"	"	"	$\wedge $	"	"	similia.
"	"	"	$\wedge \vdash$	"	"	differentia
"	"	"	$\wedge \equiv$	"	"	commensura.
"	"	"	$\wedge <$	"	"	incommensura.
"	"	"	$\vee \wedge$	"	heterical homa.	
"	"	"	$\vee \vee$	"	"	hetera.
"	"	"	$\vee $	"	"	similia.
"	"	"	$\vee \vdash$	"	"	differentia.
"	"	"	$\vee \equiv$	"	"	commensura.
"	"	"	$\vee <$	"	"	incommensura.

Let the sign, $\parallel \wedge$ indicate similitical homa.

"	"	"	$\parallel \vee$	"	"	hetera.
"	"	"	$\parallel \parallel$	"	"	similia.
"	"	"	$\parallel \vdash$	"	"	differentia.
"	"	"	$\parallel =$	"	"	commensura.
"	"	"	$\parallel <$	"	"	incommensura.
"	"	"	$\vdash \wedge$	"	"	differential homa.
"	"	"	$\vdash \vee$	"	"	hetera.
"	"	"	$\vdash \parallel$	"	"	similia.
"	"	"	$\vdash \vdash$	"	"	differentia.
"	"	"	$\vdash =$	"	"	commensura.
"	"	"	$\vdash <$	"	"	incommensura.
"	"	"	$= \wedge$	"	"	commensural homa.
"	"	"	$= \vee$	"	"	hetera.
"	"	"	$= \parallel$	"	"	similia.
"	"	"	$= \vdash$	"	"	differentia.
"	"	"	$= =$	"	"	commensura.
"	"	"	$= <$	"	"	incommensura.
"	"	"	$< \vee$	"	"	incommensural homa.
"	"	"	$< \wedge$	"	"	hetera.
"	"	"	$< \parallel$	"	"	similia.
"	"	"	$< \vdash$	"	"	differentia.
"	"	"	$\wedge =$	"	"	commensura.
"	"	"	$< <$	"	"	incommensura.

The left hand sign indicates the comparison between the terms of the propositions. Thus; in the equation, $a+b=a+b$, the sign $=$ expresses the comparison between $a+b$ and $a+b$; but if $a=b$, then the expression $a+b=a+b$ means, not only that we have an equation, but also that the existences exhibited on each side of the equation are inter se commensura i. e., each existence on one side of the equation sign, has a commensura, fellow on the same side and on the other side of the signs.

Now with the foregoing signs, we may from complete syllogisms in all the figures and modes. And commencing with the first four kinds of propositions, let two dots ($\cdot\cdot$) indicate that the existences, between which they are placed, are merely grouped together by comparison; and let AB , without dots between them mean as in Algebra, and also the signs $+$ and $-$ as in Algebra, and the following paradigms will show the plural syllogism.

PLURAL SYLLOGISM—PARADIGM 1ST.

Mode 1st.	Mode 2d.	Mode 3d.	Mode 4th.
$A..B \wedge \wedge A..B$	$A..B \vee \vee A'..B'$	$B..B \parallel \parallel B'..B'$	$A..B. \vdash \vdash C..D.$
$\parallel \parallel$	$\wedge \wedge$	$\wedge \wedge$	$\wedge \wedge$
$C..D \wedge \wedge A'..B'$	$C..D \vee \vee A'..B'$	$C..D. \parallel \parallel B'..B'$	$E..F. \vdash \vdash C..D.$
$\therefore A..B \parallel C..D.$	$\therefore A..B \vee C..D$	$\therefore B..B. \parallel C..D.$	\therefore indefinite.
Mode 5th.	Mode 6th.	Mode 7th.	Mode 8th.
$A..B \wedge \wedge A'..B'$	$A..B \wedge \wedge A'..B'$	$A..B \wedge \wedge A'..B'$	$A..B \vee \vee A'..B'$
$\wedge \wedge$	$\wedge \wedge$	$\wedge \wedge$	$\wedge \wedge$
$C..D \vee \vee A'..B'$	$C..D \parallel \wedge A'..B'$	$C..D \vdash \wedge A'..B'$	$C..D \wedge \vee A'..B'$
$\therefore A..B \vee C..D.$	$\therefore A..B \parallel C..D.$	$\therefore A..B \vdash C..D.$	$\therefore A..B \vee C..D.$

Mode 9th. $A..B \vee \vee B'..B'$ $C..D \vee \vee A'..B'$ \therefore indefinite.	Mode 10th. $A..B \vee \vee A'..B'$ $C..D \vdash \vee A'..B'$ \therefore indefinite	Mode 11th. $A..B \parallel \parallel A'..B'$ $C..D \wedge \parallel A'..B'$ $\therefore A..B \parallel \parallel C..D.$	Mode 12th. $A..B \parallel \parallel A'..B'$ $C..D \vee \parallel A'..B'$ \therefore indefinite.
Mode 13th. $A..B \parallel \parallel A'..B'$ $C..D \vdash \parallel A'..B'$ $\therefore A..B \vdash \parallel C..D.$	Mode 14th. $A..B \vdash \vdash C..D$ $E..F \wedge \vdash C..D$ $\therefore A..B \vdash \vdash E..F.$	Mode 15th. $A..B \vdash \vdash C..D$ $E..F \vee \vdash C..D$ \therefore indefinite	Mode 16th. $A..B \vdash \vdash C..D$ $E..F \parallel \vdash C..D$ $\therefore A..B \vdash \vdash E..F.$

The first paradigm shows the plural syllogism, with the first four kinds of propositions; in the following paradigm the first two and last two kinds of propositions will be combined.

PARADIGM 2D.

Mode 1st. $A..E \wedge \wedge A'..B'$ $C..D \wedge \wedge A'..B'$ $\therefore A..B \wedge C..D$	Mode 2d. $A..B \vee \vee A'..B'$ $C..D \vee \vee A'..B'$ $\therefore A..B \vee \vee C..D.$	Mode 3d. $B..B = = B'..B'$ $C..D = = B'..B'$ $\therefore B..B = = C..D$	Mode 4th. $A..B < < C..D$ $E..F < < C..D$ \therefore indefinite.
Mode 5th. $A..B \wedge \wedge A'..B'$ $C..D \vee \wedge A'..B'$ $\therefore A..B \vee \wedge C..D.$	Mode 6th. $A..B \wedge \wedge A'..B'$ $C..D = \wedge \wedge A'..B'$ $\therefore A..B = \wedge C..D.$	Mode 7th. $A..B \wedge \wedge A'..B'$ $C..D < \wedge \wedge A'..B'$ $\therefore A..B < \wedge C..D.$	Mode 8th. $A..B \vee \vee A'..B'$ $C..D \wedge \vee A'..B'$ $\therefore A..B \vee \vee C..D.$
Mode 9th. $A..B \vee \vee A'..B'$ $C..D = \vee \wedge A'..B'$ \therefore indefinite.	Mode 10th. $A..B \vee \vee A'..B'$ $C..D < \wedge \wedge A'..B'$ \therefore indefinite	Mode 11th. $A..B = = A'..B'$ $C..D \wedge = \wedge A'..B'$ $\therefore A..B = = C..D.$	Mode 12th. $\therefore A..B = = A'..B'$ $C..D \vee = \wedge A'..B'$ \therefore indefinite.
Mode 13th. $A..B = = A'..B'$ $C..D < = \wedge A'..B'$ $\therefore A..B < = C..D.$	Mode 14th. $A..B < < C..D.$ $E..F \wedge < C..D$ $\therefore A..B < < E..F.$	Mode 15th. $A..B < < C..D$ $E..F \vee < C..D$ \therefore indefinite	Mode 16th. $A..B < < C..D$ $E..F = < C..D.$ $\therefore A..B < < E..F.$

Now in mode 1st, of paradigm 1st, since $A \wedge B$, the first premise reduces to $A \wedge A$; and as $C \wedge D$ and $A' \wedge B'$, the second premise reduces to $C \wedge A'$; and hence the conclusion will be $A \parallel C$. And in mode 1st of paradigm 2d, for similar reasons, the conclusion will be $A = C$. It must also be observed that, it is their homonical relations inter se in space, which makes $A \wedge B$, while their times heterate. We have already shown heretofore, that the

homonical A to-day and the homonical A to-morrow have heterical points of time, and they may have heterical WHEREs, one to-day and another to-morrow. But for the present we will suppose that, the aggregations, with which we are about to deal, are in and continue in a state of absolute rest; then, they will not change their WHEREs in space. Now with the use of the signs already adopted, we may bring the relations of time and space into our propositions and exhibit them along with the aggregations or gregaria. Let T stand, not for time, but for times between which there may be a comparison, and let S stand for spaces in like manner; the, the proposition $A \wedge A'$, in order to exhibit the relations of times and spaces may be written thus:

$$\begin{array}{c} \vee \wedge \\ \hline \text{T. S.} \\ A \wedge A' : \end{array}$$

Which proposition may be put into common language as follows:—The homonical A, having an heterical time but an homonical space with A', is homonical with A'. And we may state the first premise of the plural syllogism in Mode 1st thus:

$$\begin{array}{cc} \vee \vee & \wedge \wedge \\ \hline \text{T.} & \text{S.} \\ \hline A..B \wedge \wedge A' ..B' . \end{array}$$

And this proposition may be stated in common language as follows:—A and B, whose times are hetera and spaces homon, having heterical times but homonical spaces with A' B', whose times are hetera but spaces homon are homonical with A' and B'. And with the signs and letters as they are now understood, as we hope, the following four propositions may be expressed:

(1.)	(2.)	(3.)	(4.)
$\vee \vee \quad \wedge \wedge$	$\vee \vee \quad \wedge \vee$	$\vee \vee \quad \vee \wedge$	$\vee \vee \quad \vee \vee$
$\hline \text{T.} \quad \text{S.}$	$\hline \text{T.} \quad \text{S.}$	$\hline \text{T.} \quad \text{S.}$	$\hline \text{T.} \quad \text{S.}$
$A..B \wedge \wedge A' ..B' .$	$A..B \wedge \vee A' ..B' .$	$A..B \vee \wedge A' ..B' .$	$A..B \vee \vee A' ..B' .$

In the above propositions, the times have reference to the temporal relations between $A..B..A' ..B'$ and the mind of the thinker, i. e., to the relations between the objective aggregations and the subjective conscious truths. And we can easily see that a rose blooming on the tree and the tree itself have an homonical time; but the rose will fade and pass away, while the tree may yet remain, and hence the times of the rose's existence and of the tree now are hetera. And the times of the aggregations exhibited in the above propositions are considered in their relations, not inter se, but to an other existence, the consciousness of the ego. But if we consider those

aggregations in their relations of time inter se without any reference to any other thing, the above propositions will be reduced to the following:

(I.)	(II.)	(III.)	(IV.)
$\frac{\wedge \wedge}{T. S.}$	$\frac{\wedge \vee}{T. S.}$	$\frac{\wedge \vee}{T. S.}$	$\frac{\wedge \wedge \vee \vee}{T. S.}$
$\frac{A.}{A.}$	$\frac{A \vee B.}{A \vee A'.$	$\frac{A \vee A'.$	$\frac{A..B \vee A'..B'.}{A..B \vee A'..B'.$

Now if we reduce the premises in modes 1st in like manner as the propositions last above given, we will have:

(a.)

1st., premise	$\frac{\frac{\wedge \wedge}{T. S.}}{A.}$	2d. premise	$\frac{\frac{\wedge \wedge}{T. S.}}{C.}$
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And by the comparison of these existences viz., A and C, we must draw the conclusion that:

(b.)	(c.)	(d.)	(e.)
$\frac{\frac{\wedge \wedge}{T. S.} \frac{\wedge \vee}{T. S.} \frac{\wedge \wedge}{T. S.}}{A \parallel C.}$	$\frac{\frac{\wedge \wedge}{T. S.} \frac{\wedge \vee}{T. S.} \frac{\wedge \wedge}{T. S.}}{A \models C.}$	$\frac{\frac{\wedge \wedge}{T. S.} \frac{\wedge \vee}{T. S.} \frac{\wedge \wedge}{T. S.}}{A = C.}$	$\frac{\frac{\wedge \wedge}{T. S.} \frac{\wedge \vee}{T. S.} \frac{\wedge \wedge}{T. S.}}{A < \text{or} > C.}$

The premises in mode 5th reduce as follows:

(f.)

	$\frac{\frac{\wedge \wedge}{T. S.}}{A.}$		$\frac{\frac{\wedge \wedge}{T. S.} \frac{\wedge \vee}{T. S.} \frac{\wedge \wedge}{T. S.}}{C \vee A.}$
And as	$\frac{\frac{\wedge \wedge}{T. S.}}{A}$	in the first premise and	$\frac{\frac{\wedge \wedge}{T. S.}}{A,}$ in the second premise

are homon i. e., $\frac{\frac{\wedge \wedge}{T. S.}}{A} \quad \frac{\frac{\wedge \wedge}{T. S.}}{A} \quad \frac{\frac{\wedge \wedge}{T. S.}}{A,}$ the comparison between C

(g.)

and A makes the conclusion	$\frac{\frac{\wedge \wedge}{T. S.}}{A}$	$\frac{\frac{\wedge \vee}{T. S.}}{V}$	$\frac{\frac{\wedge \wedge}{T. S.}}{C.}$
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The premises in modes 8 reduce as follows:

(h.)

1st., premise,	$\frac{\Lambda \vee}{\text{T. S.}}$	$\frac{\Lambda \Lambda}{\text{T.}}$	$\frac{\vee \vee}{\text{S.}}$	$\frac{\Lambda \vee}{\text{T. S.}}$
	$\frac{\Lambda \vee}{\text{T. S.}}$	$\frac{\Lambda \Lambda}{\text{T.}}$	$\frac{\vee \vee}{\text{S.}}$	$\frac{\Lambda \vee}{\text{T. S.}}$
2d, premise	$\frac{\Lambda \vee}{\text{T. S.}}$	or	$\frac{\Lambda \vee}{\text{T. S.}}$	as $\Lambda..B$ and $C..D$ are
	$\frac{\Lambda \vee}{\text{T. S.}}$		$\frac{\Lambda \vee}{\text{T. S.}}$	
	$\frac{\Lambda \vee}{\text{T. S.}}$		$\frac{\Lambda \vee}{\text{T. S.}}$	

(i.)

homonical hetero, and as	$\frac{\Lambda \vee}{\text{T. S.}}$	$\frac{\Lambda \Lambda}{\text{T.}}$	$\frac{\Lambda \vee}{\text{S.}}$	$\frac{\Lambda \Lambda}{\text{T. S.}}$
	$\frac{\Lambda \vee}{\text{T. S.}}$	$\frac{\Lambda \Lambda}{\text{T.}}$	$\frac{\Lambda \vee}{\text{S.}}$	$\frac{\Lambda \Lambda}{\text{T. S.}}$
	$\frac{\Lambda \vee}{\text{T. S.}}$	$\frac{\Lambda \Lambda}{\text{T.}}$	$\frac{\Lambda \vee}{\text{S.}}$	$\frac{\Lambda \Lambda}{\text{T. S.}}$
	$\frac{\Lambda \vee}{\text{T. S.}}$	$\frac{\Lambda \Lambda}{\text{T.}}$	$\frac{\Lambda \vee}{\text{S.}}$	$\frac{\Lambda \Lambda}{\text{T. S.}}$
therefore the conclusion,	$\frac{\Lambda \vee}{\text{T. S.}}$	$\frac{\Lambda \Lambda}{\text{T.}}$	$\frac{\Lambda \vee}{\text{S.}}$	$\frac{\Lambda \vee}{\text{T. S.}}$
	$\frac{\Lambda \vee}{\text{T. S.}}$	$\frac{\Lambda \Lambda}{\text{T.}}$	$\frac{\Lambda \vee}{\text{S.}}$	$\frac{\Lambda \vee}{\text{T. S.}}$
	$\frac{\Lambda \vee}{\text{T. S.}}$	$\frac{\Lambda \Lambda}{\text{T.}}$	$\frac{\Lambda \vee}{\text{S.}}$	$\frac{\Lambda \vee}{\text{T. S.}}$
	$\frac{\Lambda \vee}{\text{T. S.}}$	$\frac{\Lambda \Lambda}{\text{T.}}$	$\frac{\Lambda \vee}{\text{S.}}$	$\frac{\Lambda \vee}{\text{T. S.}}$

Now propositions either (3) or (III) underlies the conclusions in modes 1, 5, 6 and 7, and proposition either (4) or (IV) underlies the conclusions in modes, 2, 3, 4, 8, 9, 10, 11, 12, 13, 14, 15 and 16; for, similia, differentia, commensura and incommensura are also hetero. Our knowledge of hetero and consequently of homon depends upon time and space; but our knowledge of similia, differentia, commensura and incommensura does not depend upon time and space, but upon the gregaria of aggregations. And these substrata of our knowledge are to be inquired into from other grounds.

CHAPTER II.

SIGNS IN INDUCTION.

In heterical induction of aggregations, we find two or more instances of simillical effects, and we use one of the instances to eliminate some of the aggregations from the sine quibus non in another instance. The aggregations of the two or more instances may be synchronous or they may not be. An observation made in the time of Homer, if correctly made, is as valuable for one of the instances, as one made to-day, although the aggregations which came under observation then, may have passed away into other forms. And in making experiments, the times of the experiments are not homon but hetero. But the aggregations brought together in any one instance of an ob-

servation or experiment have homonical times. And when we view a metallic globe, for instance, of the diameter of six inches, we consider it as occupying an homonical WHERE, though the wheres of its particles be hetera. And so also, if we bring the aggregations A, B, C, D, &c., in contact with each other, we may then consider, the result as an aggregation of aggregations and as occupying an homonical WHERE. Now if we let the last letters of the Alphabet, v, x, y, z, stand for effects, and let the sign \mathcal{Z} stand for causation, then in view of what has been said above, we will have the proposition,

(1.)

$$\begin{array}{ccc}
 \frac{\wedge \wedge}{\text{T. S.}} & \frac{\vee \vee}{\text{T. S.}} & \frac{\wedge \wedge}{\text{T. S.}} \\
 \hline
 \text{A..B..C..D.} & \vee \vee & \text{A' ..B' ..C'} \\
 \hline
 \mathcal{Z} & \frac{\vee \vee}{\text{T. S.}} & \mathcal{Z} \\
 \hline
 \text{x.....} & || & \text{x'}
 \end{array}$$

And as x and x' are similical effects, they can be produced by similical hetera and in order to have similical hetera EO' NOMINE ET IN NUMERO, we must dismiss D in the first term from the sine quibus non of the effect x. We may then find another instance and have the proposition:

(1.)

$$\begin{array}{ccc}
 \frac{\wedge \wedge}{\text{T. S.}} & \frac{\vee \vee}{\text{T. S.}} & \frac{\wedge \wedge}{\text{T. S.}} \\
 \hline
 \text{A'' ..B''} & \vee \vee & \text{A' ..B' ..C'} \\
 \hline
 \mathcal{Z} & \frac{\vee \vee}{\text{T. S.}} & \mathcal{Z} \\
 \hline
 \text{x''.....} & || & \text{x'}
 \end{array}$$

And this proposition enables us to heterate C'. The heterical induction of gregaria may be represented in the same manner. Take the proposition

(2.)

$$\begin{array}{ccc}
 \frac{\wedge \wedge}{\text{T. S.}} & \frac{\vee \vee}{\text{T. S.}} & \frac{\wedge \wedge}{\text{T. S.}} \\
 \hline
 \text{A .. B} & \wedge \wedge & \text{. C .. B'} \\
 \hline
 \text{a..b..c..d e..f..g..h} & & \text{a..b..c. e..f..g..h} \\
 \hline
 \mathcal{Z} & \frac{\vee \vee}{\text{T. S.}} & \mathcal{Z} \\
 \hline
 \text{x.....} & || & \text{x'}
 \end{array}$$

Now as $B \parallel B'$, they will contain a like number of similical gregaria and hence by looking at C, d can be eliminated from the causal gregaria in A.

Homonical induction is the reverse of heterical induction. Take the proposition respecting aggregations:

(3.)

$\frac{\wedge \wedge}{T. S.}$	$\frac{\vee \vee}{T. S.}$	$\frac{\wedge \wedge}{T. S.}$
$\frac{A..B..C}{\hline}$	$\frac{\vee \vee}{\hline}$	$\frac{B'..C'}{\hline}$
\mathcal{Z}	$\frac{\vee \vee}{T. S.}$	\mathcal{Z}
$\frac{\quad}{\hline}$	$\frac{\quad}{\hline}$	$\frac{\quad}{\hline}$
x.....	+.....	0, or y.

Now as we desire to have similical effects, i. e., x and x', they must be produced by similical heteria, eo nomine et in numero, and by looking at the terms, we see that A must be added to the second term, i. e., that A was a sine qua non of the effect x.

In differential induction we first clear the way as much as possible by heterical induction of gregaria and then take the proposition:

(4.)

$\frac{\wedge \wedge}{T. S.}$	$\frac{\vee \vee}{T. S.}$	$\frac{\wedge \wedge}{T. S.}$
$\frac{A. .B}{\hline}$	$\frac{\vee \vee}{\hline}$	$\frac{C. .B'}{\hline}$
$\frac{a..b..c..d \text{ i.k. \&c.}}{\hline}$	$\frac{\quad}{\hline}$	$\frac{a..b..e..f \text{ i.k. \&c.}}{\hline}$
\mathcal{Z}	$\frac{\vee \vee}{T. S.}$	\mathcal{Z}
$\frac{\quad}{\hline}$	$\frac{\quad}{\hline}$	$\frac{\quad}{\hline}$
x.....	+.....	0, or y.

And now as $B \parallel B'$, their gregaria are similical differentia; and if $A \parallel C$, we should have had similical effects; but as the effects are differentia, their causal gregaria in A and in C are differentia: and hence the similical gregaria in A and C may be differentiated from the causal gregaria, i. e., a and b and the causal gregaria of x in A are differentia.

Similical induction is the reverse of differential induction; take the proposition.

(5.)

$\frac{\wedge \wedge}{\text{T. S.}}$	$\frac{\vee \vee}{\text{T. S.}}$	$\frac{\wedge \wedge}{\text{T. S.}}$
$\frac{\text{A.} \quad \text{.B}}{\text{a..b..c..d i..k..&c.}}$	$\frac{\vee \quad \vee}{\text{T. S.}}$	$\frac{\text{C.} \quad \text{.B'}}{\text{a..b..c..f i..k..&c.}}$
$\frac{\text{2f}}{\text{x.....}}$	$\frac{\vee \quad \vee}{\text{T. S.}}$	$\frac{\text{2f}}{\text{x'.....}}$
$\frac{\text{---}}{\text{x.....}}$	$\frac{\text{---}}{\text{ }}$	$\frac{\text{---}}{\text{x'.....}}$

Now as $x \parallel x'$, they have been produced by similital gregaria, and as $B \parallel B'$, we must find similital gregaria in A and C, and we find a and b in both; therefore these gregaria, or one of them at least is a causal gregarium.

We must notice, that in our propositions for making heterical and homonical inductions, we represent the aggregations by the signs between the terms, merely as hetera. This must necessarily be the case; for, we are eliminating and aggregating hetera by those processes. In differential and similital inductions also we must represent the aggregations by the signs, merely as hetera. For, if

$$\frac{\text{A..B}}{\frac{\text{2f}}{\text{X}}}$$

and $A \parallel B$, we know by ratiocination that similital similia will produce similital effects; and if $A \neq B$, we know that similital differentia will produce similital effects. But in the above inductive propositions, $B \parallel B'$ and $A \neq C$, as aggregations, and we desire to find in A and C, the respects, the gregaria inter se similia and to make an inference respecting them and this can be done only by using the heterical signs between the terms.

In incommensural induction, there are three cases; 1st, times and spaces being commensura, the quantities are incommensura; 2d the times and quantities being commensura, the spaces are incommensura; 3d the spaces and quantities being commensura, the times are incommensura. Let us suppose that we witness the effect in B and B', then:

(6.)

$\frac{\wedge \vee}{\text{T. S.}}$	$\frac{= \vee = \vee}{\text{T.} \quad \text{S.}}$	$\frac{\wedge \vee}{\text{T. S.}}$
$\frac{\text{A..B}}{\text{2f}}$	$\frac{< \quad \vee}{\text{T. S.}}$	$\frac{\text{A'..B'}}{\text{2f}}$
$\frac{\text{---}}{\text{x.....}}$	$\frac{\vee \quad \vee}{\text{T. S.}}$	$\frac{\text{---}}{\text{x'.....}}$
	$\frac{\text{---}}{<.....}$	

$$(7.)$$

$\frac{\wedge \vee}{\text{T. S.}}$	$\frac{= \vee < \vee}{\text{T. S.}}$	$\frac{\wedge \vee}{\text{T. S.}}$
$\frac{\text{A..B}}{\text{---}}$	$\frac{= \vee}{\text{---}}$	$\frac{\text{A'..B'}}{\text{---}}$
$\frac{2f}{\text{---}}$	$\frac{\vee \vee}{\text{T. S.}}$	$\frac{2f}{\text{---}}$
x.....	>	x'

$$(8.)$$

$\frac{\wedge \vee}{\text{T. S.}}$	$\frac{< \vee = \vee}{\text{T. S.}}$	$\frac{\wedge \vee}{\text{T. S.}}$
$\frac{\text{A..B}}{\text{---}}$	$\frac{= \vee}{\text{---}}$	$\frac{\text{A'..B'}}{\text{---}}$
$\frac{2f}{\text{---}}$	$\frac{\vee \vee}{\text{T. S.}}$	$\frac{2f}{\text{---}}$
x.....	<	x'

Commensural induction brings a simile of one of the aggregations, which we have determined by incommensural induction to contain causal gregaria of a given effect, and some other aggregation, about which we are uncertain, into relations commensural with the relations between the aggregations, which we know to contain causal gregaria of such effects. And these relations are threefold, hence:

$$(9.)$$

$\frac{\wedge \vee}{\text{T. S.}}$	$\frac{= \vee = \vee}{\text{T. S.}}$	$\frac{\wedge \vee}{\text{T. S.}}$
$\frac{\text{A..B}}{\text{---}}$	$\frac{= \vee}{\text{---}}$	$\frac{\text{C..B'}}{\text{---}}$
$\frac{2f}{\text{---}}$	$\frac{\vee \vee}{\text{T. S.}}$	$\frac{2f}{\text{---}}$
x.....	=	x'

Which proposition brings C and B' into commensural relations with the relations of A and B, and when that is done we find the commensural effect, and hence, as $B \parallel B'$, we conclude that C contains similitical gregaria with A. If we should take the second term of proposition (8,) as the first term of an inductive commensural proposition we will have:

$$\begin{array}{ccc}
 & (9.) & \\
 & = V = V & \\
 \frac{\frac{\Lambda}{T} \frac{V}{S}}{A'..B'} & \frac{\frac{\Lambda}{T} \frac{V}{S}}{C..B''} & \\
 2\epsilon & \frac{V}{T} \frac{V}{S} & 2\epsilon \\
 x' \dots\dots\dots = \dots\dots\dots x'' & &
 \end{array}$$

If we cannot thus bring the aggregations, which we are investigating, into commensural relations as above, and find commensural effects, we may, yet frequently, by mathematical calculations, find what would be the effect, if such commensural relations were realized; and this will answer the purpose.

CHAPTER III.

HETERICAL INDUCTION APPLIED.

In the two previous chapters, we have given formulæ, which, when carefully considered and fixed in the mind, will assist the understanding in investigating nature. Observations and experiments must furnish the data but the inferences to be drawn from those data must be dictated by a sound philosophy. And the formulæ, which we have given, will not only aid the mind in making proper inferences, but also in looking for the kind of instances, from which alone legitimate inferences can be drawn. And in applying the foregoing principles, it will not be necessary for us to bring the cases noticed into the exact form of the formulæ, as the reader, who has mastered the subject, can easily do that for himself. We wish merely to show the utility and importance of the subject, by illustrations from cases in which these principles have led to scientific discoveries, though the investigators, perhaps, were entirely ignorant of the processes heretofore explained. And it will not be necessary to furnish many illustrations to show what may be expected to follow from a thorough knowledge of these processes by the scientific men of the world, who are engaged in the several departments of science. Our illustrations may be taken from any department of Knowledge for our principles apply to every branch of science. We will commence with heterical induction.

Among all the varieties of material forms, which surround us in the world, chemists have been able to find fifty-five elementary substances, i. e. Substances whose particles are inter se similia. And from some or other of these elements, mineral compounds, vegetable organisms and animal organizations are produced. Now nature's laboratory can be entered, in the first instance, only by induction; we cannot commence with the simple elements and reason A PRIORI, or a posteriori, without first having made induc-

tions. There is no evidence, about which we at present know anything, to establish any belief, that what now are called elements, are really compounds; and when we find the number and kinds of elements, which, from any compound, or organization, we conclude, that we have all the *sine quibus non*, and because none other are present, i. e., by heterical induction. But because a certain number and kinds of elements are found in certain instances, or even in all instances known to us, we are not certain that each one of them is a *sine qua non* of the given effect; although this false kind or reasoning *per enumerationem simplicem* is still employed by writers upon the physical sciences.

In the organizations of animals we find an *animus* or life principle, *vis vitæ*, and this principle has been said to possess and exert a force *sui generis* upon the elements and to impart to them, when taken into the stomach, an unusual action. And although this life principle exists in all animals, yet the theory respecting its force on the elements (and it is nothing but a theory) has recently been disproved in a measure at least in the most satisfactory manner by heterical induction. It has been shown that hard boiled albumen and muscular fibre can be dissolved by adding a few drops of muriatic acid to a decoction of the stomach of a dead calf, precisely as in the stomach of a living animal. This one instance heterates the *vim vitæ* from the *sine quibus non*, and leaves the stomach to act upon chemical principles in dissolving the food; and if the known principles of chemical transformation do not yet sufficiently account for digestion, it must be further inquired into. Physiologists have also attributed the formation of formic acid, oxalic acid, urea &c., in the body to the force of the *vis vitæ*; yet each of these can be formed in the laboratory of the chemist, and consequently it is proved that *vis vitæ* is not a *sine qua non*. True heterical induction thus dispells mystic theories and opens the true road for inquiry.

Chemists have contended that vegetable fibre in a state of decay, which is called humus, is absorbed by plants and is necessary to their growth; yet this humus can be separated by heterical induction. For, although this humus is present in most soils, yet "plants thrive," as we are informed by Dr. Leibig, "in powdered charcoal, and may be brought to blossom and bear fruit if exposed to the influence of the rain and atmosphere; the charcoal may be previously heated to redness. Charcoal is the most 'indifferent' and most unchangeable substance known; it may be kept for centuries without change, and is, therefore, not subject to decomposition." Now one such case, as just cited from Dr. Leibig, who reasons more philosophically than most chemists, completely heterates the ABSORPTION of humus from the *sine quibus non*. Leibig contends further, that humus merely furnishes carbonic acid for the atmosphere surrounding the roots and stalk of the plant, and that this

carbonic acid is a *sine qua non*. This, however, cannot be proved by heterical induction, which is the only subject that concerns us at present.

We find that several kinds of opium contain maconic acid, and from the examination of such kinds alone without a true philosophy by which to test nature, we would erroneously conclude maconic acid to be a *sine qua non* of opium as an anodyne and soporific, but there are other specimens of opium, which do not contain a trace of this acid, and yet they produce similical effects. By heterical induction also, we establish the truth, that volition and the mind's command of the nervous apparatus are not *sine quibus non* of nutrition in animals. For, in those parts of the body, which have been paralyzed and which, therefore, are destitute of feeling and not subject to the mind's control, nutrition still proceeds without interruption. Oxygen may be condensed into a liquid by pressure, in which state it posess those gregaria, which distinguish a liquid from a gass; and yet in either state its actions upon other substances are *inter se similia*; and those distinguishing gregaria some in the one and some in the other state, can be heterated from the causal gregaria of the effects of oxygen. We need not illustrate further.

CHAHTER IV.

HOMONICAL INDUCTION APPLIED.

We have heretofore observed that heterical induction does not determine causes, but merely clears the way so that homonical induction can be made more easily applicable to any given case. Now we find that animals having lungs respire the atmosphere, and so long as respiration continues, the circulation of the blood and life and heat exist, but let respiration be prevented and death ensues; by homonical induction, therefore, the atmosphere is one of the causes of life and heat in such animals. And upon examination of the atmosphere, we find it to contain frequently carbonic acid, water, some earthy matters and oxygen and nitrogen. The earthy matters, carbonic acid and water can be removed from the causes of the effects of respiration by heterical induction; but if we remove the oxygen, these effects immediately cease, and hence it is certain that oxygen is a *sine qua non*. And by heterical induction we can remove all elements from the *sine quibus non* of the growth of mammalia excepting those contained in milk; for the health and growth of the young may be promoted by milk alone. Now we find milk to contain caseine, a compound containing a large proportion of nitrogen; sugar of milk, in which there are large quantities of oxygen and hydrogen; lactate of soda, phosphate of lime, common salt and butyric acid. Is each of these elements a *sine qua non*? A horse may be kept alive upon potatoes, in which the quantity of nitrogen is small, but he does not thrive, and if deprived of all food containing nitrogen, he dies. Mammalia cannot live without a salt, nor can any one of the constituents of milk be wanting

for any great length of time without a marked influence upon the health of the animal. Experiments showing such truths furnish the data for homonical inductions. Plants cannot grow if either hydrogen or carbonic acid be wanting; and hence, these are *sine quibus non*.

And again, we see that if the blood be taken from animals, the immediately die; that blood is a *sine qua non*, is therefore evident. We see also by heterical induction that food taken into the stomach is not a *sine qua non* to the life of the foetus; nor is the respiration of atmosphere; but after birth both these things by homonical induction are *sine quibus non*. Now blood is composed of fibrine and serum, and each of these has been analysed, and they are found to be isomeric, i. e., the constituents of the one and of the other are, not only similitudinal differentia, but also by weight commensural incommensura. It has been found also that if the blood be deprived of any one of its constituents, the health suffers; each one, therefore, by homonical induction is a *sine qua non*. We can prove also by homonical induction that light is a *sine qua non* of the growth and health of vegetables; for, other things being equal, they will not develop in dark cellars or caves. Most plants contain organic acids in combination with bases such as potash, soda, lime or magnesia; and hence it has been concluded, (but it is only probable and not an induction) that an alkaline base is a *sine qua non* of growth of plants. The way to prove it is to make an experiment and have all other things, found in the soil and atmosphere where the plant grows well, present excepting these bases; if the plant will then not grow, we have made an homonical induction.

In many of the sterile soils on the coast of South America, crops of grain will not grow at all; but if guano be put upon those soils, they then yield abundant crops; here is an homonical induction respecting guano. And certain soils, which are entirely barren, may be rendered fertile by putting quick lime upon them. Soils also destitute of alkalies and phosphates will not grow certain plants, but if these be added, the plants then thrive upon them; here is an homonical induction. Homonical inductions respecting the necessary constituents of soils for raising plants may readily be made by comparing a productive with a barren soil. We take the following analyses from Dr. Leibig's agricultural chemistry. A, represents the surface soil; and B the subsoil. One hundred parts contain:

	A.	B.
Silica with coarse silicious sand.....	95,843.	95,180
Alumina.....	0.600.	1,600
Protoxide and peroxide of iron....	1,800.	2,200
Peroxide of manganese.....	a trace.	
Lime in combination with silica.....	0.038.	0.455
Magnesia in combination with silica.....	0.006.	0.160
Potash and soda.....	0.005.	0.004

Phosphate of iron.....	0.198.	0.400
Sulphuric acid.....	0.002.	a trace
Chlorine.....	0.006.	0.001
Humus soluble in alkalies.....	1.000.	0.000
Humus insoluble in alkalies	0.502.	0.000
	<hr/>	<hr/>
	100,000.	100,000

The above analysed soil was characterised by its great sterility. White clover could not be made to grow upon it; it, therefore furnishes one of the cases necessary for an homological induction. In the following analysis we have:

	A.	B.
Silica and fine silicious sand.....	94,724.	97,340
Alumina	1,638.	0.806
Protoxide and peroxide of iron with manganese.....	1,960.	1,201
Lime.....	1,028.	0.095
Magnesia.....	a trace.	0.095
Potash and soda.....	0.077.	0.112
Phosphoric acid... ..	0.024.	0.015
Gypsum.....	0.010.	a trace
Chlorine of the salt.....	0.207.	a trace
Humus.....	0.512.	0.135
	<hr/>	<hr/>
	100,000.	100,000

The above soil produced luxuriant crops of lucerne and sainfoin and all other plants whose roots penetrated deeply into the ground. Now from these two cases, it would appear that in those plants receiving their nourishment from the subsoil, humus was a *sine qua non*; while gypsum is indicated as a *sine qua non* in the surface soil.

If we take muscular fibrine, which contains water, and let it be exposed to a moist atmosphere, putrification takes place; but if the fibrine be dried and then exposed to a dry atmosphere, no such result takes place. Hence water or hydrogen, is a *sine qua non* of the putrification. So also yeast, when completely dry, possesses no power to produce fermentation. Now yeast possesses a soluble and an insoluble substance, and the insoluble substance may be thrown out of the *sine quibus non* of fermentation by heterical induction; but the soluble part when exposed to the atmosphere produces fermentation, but when the atmosphere is excluded no such result takes place. An aqueous infusion of yeast may be mixed with a solution of sugar and preserved in hermetically sealed vessels without undergoing the slightest change, but if exposed to the atmosphere fermentation immediately begins. Hence the soluble part of yeast and the atmosphere are proved to be *sine quibus non* of the fermentation which ensues in such cases. Several kinds of vegetable fibre, if kept secluded from oxygen or hydrogen, do not decay, but when oxygen and hydrogen are present decay commences; each of

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these, therefore, is a *sine qua non* of such decay. Other bodies do not decay without the presence of a free alkali, and in such cases alkali by homonical induction is a *sine qua non*. The juice of grapes expressed under a receiver filled with mercury, which completely excluded the air, did not ferment; but when the smallest portion of air was admitted fermentation immediately began. Animal food and vegetables may be kept for years without fermentation, if the air be completely excluded. We have gone far enough to illustrate the manner of making and the utility of homonical inductions. Any one of the cases of induction given above may be stated in the manner of formulae (3), in Chapter II. The only difficulty in arriving at conclusions, which may be confidently relied upon, lies in obtaining the precise data needed; if these can be had our conclusions are infallible.

CHAPTER V.

DIFFERENTIAL INDUCTION APPLIED.

We have seen in the previous book, that the homonical induction of aggregations only proves a certain aggregation to have been a *sine qua non* of a particular effect, but from this case we can not infer by ratiocination that this particular aggregation or a simile of it must be a *sine qua non* of all similital effects. For, as there shown, two aggregations, as aggregations may be differentia, and yet in the respect of the gregarium, which in one of the aggregations has been a cause of the given effect, the two may be inter se similia; and hence the necessity of differential and similital inductions. This matter has been sufficiently explained heretofore. Now if we take a view of the elementary gases, we will see by differential induction, that those gregaria, which distinguish gasses from liquids and solids, are not the causal gregaria of the peculiar action of any gass upon another substance; for, in these distinguishing gregaria gasses all agree. By differential induction we know, that the peculiar action of oxygen upon iron, for instance, is not owing to the distinguishing gregaria of a gass; for if it were, nitrogen would produce upon iron a similital effect. The chemical action of liquids and of solids may be treated in a like manner. Each element possesses a chemical gregarium sui generis; and by differential induction we may frequently draw so near to this gregarium, which is a cause of certain effects, as to leave no doubt of the causal gregarium, though differential induction does not directly determine causes. Complete differential inductions of all the elements would lay the foundations upon which chemistry might be made a deductive science; which may, as we hope, be accomplished in the future. But for the illustration of our present subject, we must proceed with such data as experimentalists have furnished. And we may commence, not with the differential induction of elements, but of compounds. One element may not exert some peculiar force without the presence of another or others, with

which it is compounded, and then this peculiar compound is the sine qua non of a given effect. This is owing to the circumstance, that compounds possess capacial gregaria, which, with reference to the gregaria of either of the elements entering into them, are differentia. We may begin our illustrations, therefore, by differentiating compounds. And as by analysing composite substances, they are resolved into simple differential compounds, we may assume, for the sake of illustration, that each of the simpler compounds, into which a composite substance can be resolved, exerts its gregaria unimpeded when in the more complex substance.

Now according to Brandes, rhubarb contains: Rhubarbic acid; Galic acid; Tannin; Sugar; Colouring extractive; Starch; Gummy extractive; Pectic acid; Malate of lime; Gallate of lime; Oxalate of lime; Sulphate of pottassa; Chloride of pottasium; Silica; Phosphate of lime; Oxide of iron; Lignin; Water. And if by differential induction we are in search of the purgative ingredient of rhubarb, we may differentiate water by a comparison of rhubarb with the juice of the sugar cane, both contain water, they agree in this respect; we may differentiate lignin by a comparison with almost any woody fibre; the oxide of iron and silica by a comparison with the water from wells and thermal springs; phosphate of lime by a comparison with bone dust chloride of pottassium by a comparison with sea-water; the sulphate of pottassa by a comparison with potashes; the oxalate of lime by a comparison with wood-sorrel; Gallate of lime by a comparison with gall-nuts; the malate of lime by comparison with vegetables such as the house-leek; Tannin by a comparison with the bark of oaks; sugar and starch by comparisons with wheat flour and maple saps &c. As the above compounds can be separated, we could use heterical and homonical inductions, and that is the better way, for, it relieves us from making an assumption at the outset which may not be true; but for the sake of illustration we have used differential induction. If we wish to find by differential induction in what the poisonous gregaria of morphia consist, we may analyze this compound and we find it to contain carbon, hydrogen, oxygen and azote. We can differentiate the carbon by the comparison with fat beef or pork; the hydrogen and oxygen by a comparison with water; and the azote by a comparison with gluten or indigo. And hence it appears that neither of these elements per se is the cause of the poisonous effects of morphia, but that the causal gregaria arise from the compound. There is in this induction, however, the same assumption, which we made, when, treating of rhubarb, and though, we think, we are at liberty to make such assumption for the sake of conveying to the reader's mind the application of a principle yet in the actual search after truth, such assumption is inadmissable. We must deal with morphia, therefore, not by its ingredients, but by its gregaria.

Now morphia among others coontains the following gregaria.

MORPHIA.—It is fusible at moderate heat; it burns with a red and very smoky flame; it is soluble in 30 parts of boiling anhydrous alcohol; it is soluble in 500 parts of boiling water; it is insoluble in cold water; it is insoluble in ether; it is insoluble in oil; it is insoluble in chloroform; it forms salts with acids.

We will assume that the above data are correct, though chemists differ respecting some of the gregaria. The following are some of the gregaria of starch a non-poisonous substance:

STARCH.—It is insoluble in cold water; it is insoluble in cold alcohol; it is insoluble in ether; it is insoluble in oil.

The following are some of the gregaria of resin a non-poisonous substance.

RESIN.—It is fusible at moderate heat; it is insoluble in water; it is translucent; it burns with bright flame and very much smoke.

Now, if we compare morphia with these last two non-poisonous substances we will see that several of their gregaria are inter se similia; these gregaria, therefore, may be differentiated from the poisonous gregaria contained in morphia, and further investigation must be had.

Again; we know that common salt, chloride of sodium, is an antiseptic and when applied to fresh flesh it prevents decay; we may inquire therefore, respecting the causal gregarium of this phenomenon. Now among the gregaria of common salt are the following:

SALT.—It has a white color; it has a saline taste; it undergoes but little change in a dry atmosphere; it dissolves in water; it dissolves but little in alcohol; it melts by heat; it is decomposed by carbonate of potassa.

With common salt we may compare Epsom salts, sulphate of magnesia, which among others contains the following gregaria:

EPSOM SALTS.—It has a white color, it has a saline taste, it undergoes but little change in a dry atmosphere, it dissolves in water, it dissolves but little in alcohol, it melts by heat, it is decomposed by carbonate of potassa.

Now the similia may be differentiated from the causal gregaria and the matter must then be further inquired into. We have gone far enough with our illustrations to see that true differential inductions can be obtained only from the comparison of gregaria. And any one who will examine the matter, will find, that in what Bacon would call the history of substances, chemical science is yet very defective. We need further experiments to be made under the guidance of a true philosophy.

CHAPTER VI.

SIMILICAL INDUCTION APPLIED.

As heterical induction clears the way for homonical induction, so differential induction prepares the way for similical induction. And both differential and similical inductions to be satisfactory must be based upon a great number of gregaria, which requires a very extensive knowledge of

substances. We do not propose to give complete and wholly satisfactory inductions respecting the causal gregaria of effects; for that would require a different kind of treatise from the one upon which we are now engaged, but we merely propose to illustrate the principles of induction and let scientific men, each one in his own special department, make the application with full data to particular subjects. Now if we wish to find the poisonous gregarium or gregaria contained in Muriatic or Sulphuric acid, we may examine their gregaria in the following manner. Some of the gregaria of Muriatic acid are as follows:

MURIATIC ACID.—It is a colorless liquid, it has a sour taste, it corrodes animal tissues, it is incompatible with metallic oxides, it is incompatible with alkalies, it redens litmus paper, it has a strong affinity for water.

The following are some of the gregaria of Sulphuric acid:

SULPHURIC ACID.—It is a colorless liquid, it has a sour taste, it corrodes animal tissues, it is incompatible with metallic oxides, it is incompatible with alkalies, it redens litmus paper, it has a strong affinity for water.

For the purpose of differential induction we may compare with the above acids the acetic acid of commerce, a substance which may be taken in large quantities without poisonous effects. Some of the gregaria of acetic acid are as follows:

ACETIC ACID.—It is a colorless liquid, it has a sour taste, it is incompatible with metallic oxides, it is incompatible with alkalies, it redens litmus paper, it has a strong affinity for water.

Now if we differentiate the similitical gregaria of acetic acid from the poisonous gregaria contained in sulphuric and muriatic acids, we find the latter two acids to agree in their gregaria of corroding animal tissues. And by similitical induction this corroding gregarium is a causal gregarium of the poisonous effects; it produces the direct destruction of the organs with which it comes in contact, and hence death ensues.

There is another class of poisons, which do not corrode or immediately destroy the organs with which they come in contact, but by their action they render the tissues incapable of performing their functions. Of these we may compare the salts of lead and of copper.

The following are some of the gregaria of the carbonate of lead:

CARBONATE OF LEAD.—It is a white solid; it is insoluble in water, it is soluble in acid, it is soluble in alkali, it enters into firm combination with animal tissues.

The following are some of the gregaria of what is commonly called verdgris, the carbonate of copper:

CARBONATE OF COPPER.—It is a green solid, it is insoluble in water, it is soluble in acid; it is soluble in alkali, it enters into firm combination with animal tissues.

For purposes of differential induction we may compare pure indigo with the above:

INDIGO.—It is a blue solid, it is insoluble in water, it is soluble in acid, it is soluble in alkali.

After differentiating we find carbonate of lead and copper to agree in the gregarium of entering into firm combination with animal tissues; and vital organs thus rendered calous and inflexible can not, of course, perform their functions, and hence death must ensue. We do not, however, give the above as satisfactory inductions; the data are insufficient and some of them may not be correct. Chemists have not been familiar with the inductive processes and they have not looked for data in view of making differential and similkal inductions, and hence they have not furnished us with the requisite ground-works.

As another case to illustrate the principle of similkal induction we may inquire into the causes of the double refraction of light. Some of the gregaria of the carbonate of lead, which substance causes double refraction, are the following:

CARBONATE OF LEAD.—It is a transparent substance, it is of crystalline structure, its crystals are of the rhombohedral form, it is insoluble in water, it is soluble in acid, it is soluble in alkali.

The following are some of the gregaria of Iceland spar, another substance causing double refraction:

ICELAND SPAR.—It is a transparent substance, it is of crystalline structure, its crystals are of the rhombohedral form, it is insoluble in water, it is soluble in acid.

The following are some of the gregaria of one species of diamond, which causes double refraction:

DIAMOND.—It is a transparent substance, it is of crystalline structure, its crystals are of the rhombohedral form, it is insoluble in water, it is soluble in acid.

With the foregoing double refracting substances we may compare the following substances, which do not refract light in that manner. The following are some of the gregaria of a species of diamond which causes single refraction:

DIAMOND.—It is a transparent substance, it is of crystalline structure, its crystals are of the octohedral form, it is insoluble in water, it is soluble in acid.

The following are some of the gregaria of pure rock salt:

ROCK SALT.—It is a transparent substance, it is of crystalline structure, its crystals are either of the cubical or octohedral form but sometimes prismatic, it is insoluble in water, it is soluble in acid.

The following are some of the gregaria of pure borax:

BORAX.—It is a transparent substance, it is of a crystalline structure, its crystals are either of the prismatic or octohedral form.

Now after using differential inductions we find the substances causing double refraction to agree in having their structure made up of rhombohedral

crystals. And from this it would appear that the form of the crystal causes double refraction; but our data are again insufficient for a satisfactory induction. There are fourteen different forms of crystals entering into the structure of diamonds and only two of which, the octohedra and cube, so far as we can learn, cause single refraction. The subject needs further examination with more full and more certainly correct data. Fesnel explains, deductively, double refraction by assuming that the ether in double refracting substances is not equally elastic in all directions. This is, of course, merely an hypothesis, and the evidence by which it can be inductively proven is not furnished by double refracting substances. Newton concluded, probably *per enumerationem simplicem*, that combustibility was in some way a cause of refraction and then reasoning *A POSTERIORI* he conjectured that water and the diamond would be found to contain combustible elements; and his conjecture has been verified. But we have gone far enough to illustrate the principle of similitudinal induction.

CHAPTER VII.

INCOMMENSURAL INDUCTION APPLIED.

We have seen, heretofore, that there are three cases of incommensural induction, having reference to three kinds of relations between the causes and their effects. And if we commence our illustrations with incommensural quantities of certain objects, which we are examining for the purpose of determining their relations to certain incommensural effects, we will soon see the utility of this method from the daily necessities of life. On making our fires in the stove, we need but admit a small current of air and then a greater one to convince us, by incommensural induction, that the atmosphere is connected, in some manner through causation, with the combustion going on in the stove. And we need but increase the inhalation of oxygen into our lungs to find out, that certain phenomenal effects in our system are dependent upon the respiration of this gas. The incommensural quantities of the sun's rays falling vertically and obliquely upon equal areas in different latitudes, must also convince us of their relations through causation with the earth's temperature and vegetation. And in every branch of agriculture, horticulture and floral training, the case of incommensural inductions from the relations of quantity may be made by a little ingenuity. I extract the following facts from Prof. Liebig's agricultural chemistry: "The employment of animal manure in the cultivation of grain and the vegetables which serve for fodder to cattle, is the most convincing proof that the nitrogen of vegetables is derived from ammonia. The quantity of gluten in wheat, rye and barley, is very different; these kinds of grain also, even when ripe, contain this compound of nitrogen in very different proportions. Proust found French wheat to contain 12.5 per cent. of gluten; Vogel found that the

Barbarian contained 24. per cent; Davy obtained 19. per cent. from winter and 24. from summer wheat; from Sicilian 21. and from Barbary wheat 19. per cent. The meal of Alsace wheat contains, according to Boussingault 17.3 per cent. of gluten; that of wheat grown in the 'Jardin des Plantes' 26.7 and that of winter wheat 3.33 per cent. Such great differences must be owing to some cause, and this we find in the different methods of cultivation. An increase of animal manure gives rise not only to an increase in the number of seeds, but also to a most remarkable difference in the proportion of the substances containing nitrogen, such as the gluten which they contain. * * * * * One hundred parts of wheat grown on a soil manured with cow-dung (a manure containing the smallest quantity of nitrogen) afforded only 11.95 parts of gluten and 64.34 parts of amylin or starch; while the same quantity grown on a soil manured with human urine, yielded the maximum of gluten, namely 35.1 per cent. Putrified urine contains nitrogen in the forms of carbonate, phosphate and lactate of ammonia and in no other form than that of ammonical salts." Now, in the above facts, granting the soils and atmospheres to have been in all other respects inter se simillal and commensural, there is a fair incommensural induction respecting ammonia.

In another case of incommensural induction, we have seen that, *ceteris paribus*, the spaces between an object containing causal gregaria and the incommensural effects are incommensural; and we will now proceed to give a few simple illustrations of this case. It is said that Galileo, perceiving that the chandeliers suspended in a church, when set in motion, vibrated long and with uniformity, was led by these phenomena to invent the pendulum. With this instrument a great many persons have since experimented; and the phenomena of its vibrations are found to be incommensura in different latitudes and localities. A pendulum of about 39 inches, which vibrates seconds in the latitude of New York, will not vibrate sixty times in an hour of commensural time on the equator; and there is a marked difference in the time of the vibrations of the same pendulum in the valleys of the Amazon and on the high peaks of the Andes. The farther you remove the pendulum from the earth's center of gravity, the fewer will be its vibrations, *ceteris paribus*. And hence we learn from these incommensural relations of spaces between the earth and the incommensural effects, that the earth contains causal gregaria of these phenomena. Again: The surveyor, from the incommensural relations of spaces between his compass and a certain hill and incommensural variations of the needle from the true meridian, concludes that the hill possesses causal gregaria of these variations. The incommensural relations of the spaces, between the moon and the waters on different parts of our earth, and the tides, furnish also the data from which to make incommensural inductions; and although the tides, on the opposite side of the earth from the moon, might seem at first thought, to destroy the force of these data, yet when

we reflect that the earth is interposed between the moon and those tides, the data remain in their validity. The reader will understand that in incommensural induction from incommensural relations of space, we are seeking merely for some object which contains causal gregaria of the incommensural effects; no matter what may be the characters, in other respects, of the incommensural effects in their relations inter se. Thus: if we try the positively electrified end of a cylinder with the knob of a charged Leyden jar and find the cylinder to be repelled, and then we try the negative pole of the cylinder and find phenomena of an opposite character, by incommensural relations of space and the incommensural effects of each kind inter se, i. e., incommensural similia both these sets of phenomena, though inter se differentia, are proved to have a dependence upon the knob of the jar, i. e., the knob contains causal gregaria of both these sets of phenomena.

We will now give a few illustrations of the case in which incommensural inductions can be obtained from incommensural relations of times. If we should find by the side of a mountain a ledge of iron ore which had been uncovered for but a quarter of a century, and on the same mountain we should find ore, which had been bare for several centuries, and we should make comparisons between the two, we would be able to draw, from the incommensural effects perceived in the ores, conclusive incommensural inductions of the cause from the incommensural relations of times, had we never thought of the cause before. For, granting that all other things are simillal and commensural in the two sets of phenomena excepting the times of exposure to the atmosphere, and the quantities of atmosphere being commensura in commensural times, no object whatever, excepting the atmosphere could have incommensurated the effects witnessed in the oxydized ores. A hound by instinct as we call it, makes a kind of inverse incommensural induction concerning incommensural effects from incommensural relations of time, or we, at least, may make it for him, when he is pursuing the trail of a deer. Each tread of the deer deposits in the soil a certain effect, and these effects immediately after the treads in simillal soils are, no doubt, very nearly commensural inter se, and which the atmosphere with the soil commences to diminish, leaving at incommensural intervals of time from the point from which they were made incommensural effects. When the hound, therefore, strikes a rather old track, not having a scientific knowledge of the relations of time, space and velocity, and no means, in the present case, of judging of the last, he is not very animated in the pursuit, not expecting to find the deer for some time, although it may have lain down within forty rods from the point where he struck the trail. But as he moves on, he perceives incommensura; he then increases his speed, and finding the degrees of the incommensura, or differences, to increase rapidly, he becomes warm and boisterous, proclaiming as he goes the state of his expectations, in relation to time of coming up

with the cause of these incommensural phenomena. Should a man buy two pair of boots *inter se similia*, and walk in one pair over a given road for six hours a day for two months, and then in like place and manner walk in the second pair for four months, and observe the incommensural effects and times, he would not hesitate to make an incommensural induction. We need go no further with illustrations.

It must have been noticed by the reader that when we are considering incommensural effects *inter se*, our comparisons have reference to nothing else than quantity, i. e., the effects *inter se* are quantitatively incommensura. It will be noticed too, that drops of water *inter se* commensural falling at intervals of one second for one year, and commensural drops falling in like manner for half a year, produce incommensural effects from the incommensural quantities of cause. And when an aggregation exerts from itself influences through space, as in the radiation of heat for instance, an object nearer and one more remote from the focus of influence, providing the objects be *inter se* commensura, will receive incommensural quantities of the influence in commensural times. And hence, laying aside the interference of causes, the quantities of causes and effects are proportional. The assertion that effects are proportional to their causes, however, must not be understood to mean that such is the case absolutely and without limit, as we will better understand hereafter.

CHAPTER VIII.

COMMENSURAL INDUCTION APPLIED.

Commensural like incommensural induction deals only with effects, which are *inter se similia*. And we take a certain case, in which we have heretofore determined a certain object to contain causal gregaria of a specific effect, and having determined the time space and quantity in this case, we endeavor to ascertain what objects, over which we may have no control, contain similital gregaria with reference to such similital effects, from the relations of the time, space and quantity of the case in which the object is under our control to the time, space and quantity in other cases of similital effects in which the objects containing causal gregaria are not under our control. And in commensural as in incommensural induction there are three cases. Let us commence our simple illustrations with the commensural relations of space. Suppose, for instance, we had made experiments with a certain ivory ball and found that when we let this ball fall forty feet upon iron of a smooth surface, it rebounded a certain number of feet; when we let it fall upon marble in like manner it rebounded a certain other number; and when upon brass in like manner a certain other and so on: and in all these experiments we will suppose the plates of the different metals and minerals with which we experimented to be quite thick and placed upon solid granite rock. The

rebounding of the ball is the effect in the ball witnessed by us, of which the space through which it rebounds is the quantum: and some of the causal gregaria of this effect are in the ball and the others are in the objects upon which it fell. Suppose now, after this, we find a mass of metal, of a kind unknown to us, underlain with granite and we let the same ivory ball fall upon its smooth surface forty feet and observe its rebounding, and we find this effect to be commensural with that obtained when it was let fall upon marble; then as the ball is the same and other things are equal, the commensural relations of the spaces fallen through by the ball in the two cases to the commensural effects, convince us by commensural induction, that this new metal contains, in the respect to these similical and commensural effects, similical and commensural causal gregaria with those contained in marble. And should this new metal be so situated that, we could not approach to it so as to examine it closely with our eyes or feel it with our hands and the ball used be an heterical one, but similical and commensural with the first, the result would be the same. Again: Suppose we make experiments with a certain magnet and find that if we attach the one end of a small string to the north pole of a magnetic needle placed at a certain distance from the magnet and the other end to a weight, which the magnet, when the magnetic needle is at right angles to it, will just be able to draw on a certain surface until the needle points directly towards the magnet, this drawing of the weight then may be taken as the quantum of the effect: if we now take a piece of ore and situate the needle with weight attached on the same surface as before, and a commensural effect be produced, we conclude by commensural induction, having our eye on the commensural relations of the spaces in the two cases and times being supposed commensural, that the magnet and ore contain similical and commensural causal gregaria. Again; if we make a fire in a stove and hold a thermometer at a certain distance from it and read the degrees to which the mercury rises in a given time, this rising of the mercury will be the quantum of the effects; if then we go to a heap of quick lime with water thrown upon it and covered up with earth, and we place the thermometer at a commensural distance from it and find the quanta of effects to be inter se commensural, we conclude that the heap contains similical and commensural gregaria, respecting such effects, with the stove.

Second Case.—If we take the down of the goose and find that a certain quantity will be attracted through a certain space in a given time by the prime conductor of an electrical machine, and we then take a commensural quantity of the down of the swan and find it to be attracted through the same space in a commensural time, we conclude the latter substance to contain similical and commensural causal gregaria with the former. If a weight be attached to a balloon and the balloon then ascend a given distance in a certain time, and we then attach the same weight to another commensural balloon

and the second one make the same distance in a commensural time, the two balloons contain similical and commensura gregaria.

Third Case.—If we charge a certain Leyden Jar to its capacity and measure the space through which a spark from the knob can be made to pass so as to ignite sulphuric ether and then we discharge a spark of the jar commensurably charged through the same space into ether of alcohol and find commensural effects, times being equal, we conclude the two ethers to contain similical and commensural causal gregaria with reference to such effects. We need not illustrate farther. If the reader will bear in mind that all effects are produced by heterical causal gregaria, some of which are in the objects in which we witness the effect, and some in another object, numerous examples, from which commensural inductions can be made, will suggest themselves to his own mind. And it is evident that if we can not always find commensural relations, we may yet make our inductions in many cases by the commensural relations of mathematical ratios. By taking a piece of iron, for instance, to incommensural distances from the earth's surface and finding the ratios of its weights and distances, we find that gravity varies inversely as the square of the distance; we find also that the matter tends to move in straight lines with a force equal to its weight multiplied into its velocity; and therefore, near the surface of the earth if we project a stone of a certain weight in a horizontal direction with a given velocity, we can calculate the distance it will make through space in falling to the earth by gravity. Now if we contemplate the moon and find its ratios to be commensural with the ratios of our experiment with the stone, we conclude by commensural induction, that the moon and the stone contain similical causal gregaria. In this manner Newton extended gravity to the moon, and it has since been extended to other heavenly bodies; and it is supposed, by *inductio per enumerationem simplicem*, to exist throughout the universe.

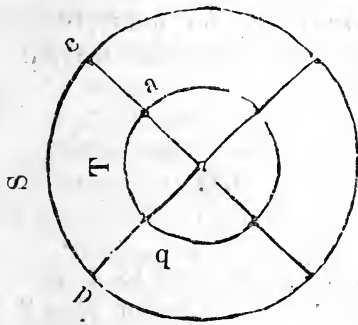
CHAPTER IX.

THE DENOMINATE UNIT.

Those who have mastered the principles of books I and II, and of the previous chapters in this book, (which in the last four chapters we have endeavored to render more easy for the understanding by giving simple illustrations with sensuous objects) will be able now to proceed further with us in our still deeper inquiries into nature's processes. In our previous inquiries, except in similical and differential inductions, we have dealt mostly with aggregations, and have not given much of our attention to gregaria, from which only, those relations, which are called the laws of nature, can be evolved. And we have seen, heretofore, that *homon per se* makes no part of our knowledge, but that we gain our knowledge of *homon* by means of *hetera*; but our knowledge of *similia* and of *differentia* is not predicated upon *hetera* alone,

but upon similitical and differential relations of gregaria; and if we can deal with these gregaria so as to discover the laws of causation by which they act, we will have to enter nature's mysteries in this regard by getting hold of relations existing inter gregaria. Now, nature is more accessible in some points than others, and her relations of quantities are most easily comprehended by us; we will, therefore commence to evolve the laws of gregaria by investigating their quantitative relations. But for this purpose we need denominate numbers, which have an homonical standard of measure; and space is the only thing from which we can gain such denominate and homonical unit. We will, therefore, treat briefly of the denominate unit in this chapter.

If the hand of a clock, when it ticks once, passes from a to b (Fig. 1.)



in the small circle of the diagram, while a body on the larger circle passes from c to d, we may take the well known equation in natural philosophy

$$V = \frac{S}{T}$$

in which relations the space from a to b may be made the denominate and homonical unit of measure; and if this unit will apply twice to the space from c to d then

$$V = \frac{2}{1} = 2.$$

The space from a to b may be made also the homonical unit of measure for a steelyard, a barometer, a thermometer, steamgauge, momentum, dry measure, liquid measure, money and throughout nature.

Then let V stand for velocity, S for space, T for time, W for weight, and M for momentum, and take the following equations in natural philosophy:

1.	2.	3.	4.	5.	6.
$V = \frac{S}{T}$	$S = VT$	$T = \frac{S}{V}$	$W = \frac{M}{V}$	$M = VW$	$V = \frac{M}{W}$

Now if V in equations 1, 2 and 3 be equal to V in equations 4, 5 and 6 as it may be, and we take the value of V as given in equation 6 and put it for V in equations 1, 2 and 3; and we take the value of V as given in equation 1 and put it for V in equations 4, 5 and 6, we will have the following equations:

7.	8.	9.	10.	11.	12.
$\frac{M}{W} = \frac{S}{T}$	$S = \frac{MT}{W}$	$T = \frac{SW}{M}$	$W = \frac{MT}{S}$	$M = \frac{SW}{T}$	$\frac{S}{T} = \frac{M}{W}$

Gravity, in a body above the earth's surface, is nothing else than the tendency of the aggregation to fall to the earth, and the quantum of space occupied by incommensural aggregations inter se similical, which is found by multiplying together their lengths, breadths and thickness, is in proportion to the quantum to this tendency to fall. If we take two pieces of lead inter se similical, but occupying incommensural spaces, the piece occupying the greater quantum of space at commensural distances from the earth's center of gravity will possess a greater quantity of gravity than the other. Now by experiments it has been ascertained, that gravity above the earth's surface, varies inversely as the square of the distance from the earth's center, or directly as the ratios obtained by dividing the square of the radius by the square of the distance from the earth's center to the body above the earth's surface. And hence let G stand for gravity, r for radius, Q for quantity of matter, and S for the distance of the body from the earth's center, and we will have the following equations:

$$13. \quad G = \frac{Qr^2}{S^2}$$

$$14. \quad Q = \frac{S^2G}{r^2}$$

$$15. \quad S^2 = \frac{Qr^2}{G}$$

Now if S in equations 1, 2 and 3, be equal to S in equations 13, 14 and 15, as it may be, and we substitute the value of S as given in equation 2 for S in equations 13, 14 and 15, and the value of S as given in equation 15 into equations 1, 2 and 3 we will have:

$$16. \quad G = \frac{Qr^2}{V^2T^2}$$

$$17. \quad Q = \frac{V^2T^2G}{r^2}$$

$$18. \quad V^2T^2 = \frac{Qr^2}{G}$$

$$19. \quad V = \frac{r}{T} \sqrt{\frac{Q}{G}}$$

$$20. \quad S = RT \sqrt{\frac{Q}{G}}$$

$$21. \quad T = \frac{R}{V} \sqrt{\frac{Q}{G}}$$

Now what is called the specific gravity of bodies, i. e., the relation of gravities between a certain quantity of water or air, and a commensural quantity of differential substances as measured by space, varies directly as the ratio obtained by dividing the gravity of a certain substance by the gravity of a commensural quantity of water or air. Hence let Q stand for the commensural quantity of any substance, 1 for the gravity of a quantity of water equal to Q , and G for the gravity of Q in any other substance than water and S for specific gravity, and we will have the following equation:

22.

$$S = \frac{G}{1}$$

And if G in equation 22 equal G in equation 16, and we substitute we will have:

23.

$$S = \frac{Qr^{21}}{V2T^2}$$

And in all the foregoing equations the standard of measure is a denominate and homonical unit of space.

CHAPTER X.

RATIO.

If one of two numbers be made the numerator and the other denominator of a common fraction, the ratio of the numerator to the denominator is such number, that if you multiply the denominator by it you will have the numerator, and if you divide the numerator by it you will have the denominator; and the ratio of the denominator to the numerator is such number, that if you multiply the numerator by it you will have the denominator, and if you divide the denominator by it you will have the numerator; and as the ratio of two numbers generally appears in the form of a fraction, (which however, may sometimes be a whole number) when you have the ratio of the numerator to the denominator, if you invert the terms of the fraction, you will have the ratio of the denominator to the numerator, and vice versa. Now all persons, who have studied mathematics, will understand the following propositions:

$$a \times 0 = 0. \quad \frac{0}{a} = 0. \quad \frac{a}{0} = \infty. \quad \frac{a}{\infty} = 0. \quad \frac{\infty}{\infty} = 1. \quad \frac{\infty}{0} = \infty. \quad \frac{0}{\infty} = 0. \quad \infty \times 0 = 1. \quad \text{and} \quad \frac{0}{0} = 1.$$

In these propositions zero or 0, is to be understood as meaning an infinitesimal quantity, i. e., a quantity less than any assignable quantity and ∞ is its reciprocal.

Now none of the foregoing propositions, excepting the last one, need any explanation for the mathematicians; the symbol

$$\frac{0}{0}$$

however, needs some explanation as the mathematical treatises used in our schools and colleges have not given to it its true significance, which we will now proceed to explain. Take the proposition

$$1. \quad x = \frac{a^3 - b^3}{a^2 - b^2}$$

If in this equation we make $a=b$, we will have

$$2. \quad x = \frac{0}{0} = 1.$$

But in equation 1 the numerator is a multiple of $a-b$, and it may be put into the form of $(a-b)(a^2+ab+b^2)$; and the denominator is also a multiple of $(a-b)$, and it may be put into the form of $(a-b)(a+b)$, and then we will have

$$3. \quad x = \frac{(a-b)}{(a-b)} \times \frac{(a^2+ab+b^2)}{(a+b)}.$$

Now from this equation we may have

$$4. \quad x = \frac{0}{0} \times \frac{(a^2+ab+b^2)}{(a+b)} = \frac{0 \times a^2 + 0 \times ab + 0 \times b^2}{(0 \times a + 0 \times b)} = \frac{0}{0} = 1:$$

Or we may have

$$5. \quad x = \frac{0}{0} \times \frac{(a^2+ab+b^2)}{(a+b)} = 1 \times \frac{(a^2+ab+b^2)}{(a+b)} = \frac{3a^2}{2a} = \frac{3a}{2}.$$

How are those incommensural results to be explained? Now as

$$\frac{0}{0} = 1,$$

1 is the ratio of the numerator to the denominator and also of the denominator to the numerator, as it always is when the numerator and denominator are absolutely commensura; thus

$$\frac{0}{0} = 1, \frac{4}{4} = 1, \frac{8}{8} = 1, \text{ etc.}$$

And it is evident that in equation 4 we have taken the fraction

$$\frac{a^2+ab+b^2}{a+b}$$

and multiplied its numerator and denominator by the common infinitesimal quantity

$$\frac{a}{\infty}$$

while in equation 5 we have multiplied the same fraction by the ratio of

$$\frac{a}{\infty} \text{ to } \frac{a}{\infty}$$

i. e., by the ratio of commensural quantities. Now it is evident that when the numerator and denominator of a fraction are commensura, their ratio will be the denominate unit, and it is also evident that, in all proper fractions the ratio of the numerator to the denominator will be less than the denominate unit: it is also evident that the difference between $\frac{1}{4}$ and $\frac{1}{2}$ will be a greater quantity with reference to the denominate unit, than the difference between $\frac{1}{8}$ and $\frac{1}{4}$, while their ratios are commensura: thus $\frac{1}{4} \div \frac{1}{2} = \frac{1}{2}$ and $\frac{1}{8} \div \frac{1}{4} = \frac{1}{2}$ and $\frac{1}{2} = \frac{1}{2}$; but $\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$ and $\frac{1}{4} - \frac{1}{8} = \frac{1}{8}$ and $\frac{1}{4} > \frac{1}{8}$. And the greater the decrease of the numerator and denominator, while their ratios remain commensura, the less will be their difference in numerical value compared with the denominate unit; and hence the difference between the numerator and denominator may become infinitesimal and the ratio all the time remain the same, i. e., $0 - 0 < 0$, while $0 \div 0 = 1$, results, which can only be true of infinitesimal quantities in their relations to our minds. And if by 0 we mean absolutely nothing at all, $0 \div 0$ is nothing, $0 - 0$ is nothing and 0×0 is nothing; and if by ∞ we mean something without limit, $\infty \times \infty$ is not within our conceptions, nor is $\infty \div \infty$. But although we can not conceive of absolute existences, and of course can not deal with them intelligently, yet we can conceive of finite relations as being absolutely commensural and incommensural and hence if we have equation 4 or 5 as above, we may con-

ceive of the relations of $a-b$ in the numerator and in the denominator as absolutely commensural, and of a and b as absolutely commensura, and then the relations contained in

$$\frac{a-b}{a-b}$$

$$a-b$$

will destroy each other and this fraction will have no relation to offer towards the other factor, i. e., its relations will be a nonentity and it need not be considered, but if $a-b$ in the denominator be an infinitesimal quantity and $a-b$ in the numerator be an absolutely commensural infinitesimal quantity,

$$\frac{a-b}{a-b}$$

$$a-b$$

will absolutely equal 1, the denominate unit; and we have seen in the previous chapter, that the denominate unit is the space which is the homonical standard for the measurement of time. Now whenever any number is multiplied by 1 the number is taken one time, i. e., its value is not affected; and whenever a number is multiplied by absolutely nothing, i. e., not touched at all, its value is not affected; and hence any number multiplied by absolutely nothing will remain in the same relations, as when it is multiplied by the ratio of two numbers, whose difference is absolutely nothing; and therefore in equation 4 we multiplied both numerator and denominator by an infinitesimal quantity, which produced products whose difference was not absolutely nothing though taken to be so, while in equation 5 we multiplied by the ratio of two numbers, whose difference was absolutely nothing, and hence the incommensural results. And upon the supposition with which we started, i. e., that a was absolutely equal to b , equation 5 contains the true result.

Now from the foregoing discussion it will appear that, the symbol $\frac{0}{0}$ may be made to make its appearance in every ratio by factoring and supposing the difference between the numerator and denominator of one of the factors to be less than any assignable quantity: thus the ratio of 4 is $\frac{1}{2}$, which may be equal to $\frac{a}{b}$

$$\frac{a}{b} \times \frac{1}{2},$$

$$\frac{4}{8}$$

when the difference between a and b is less than any assignable quantity and we multiply by their ratio; but if we multiply by the quantities themselves we will have

$$\frac{0}{0}$$

$$\frac{0}{0}$$

i. e., we will have 1 instead of $\frac{1}{2}$. To illustrate by figures let $\frac{1}{2} = \frac{4}{4} \times \frac{1}{2}$

and if $4=4$ absolutely and we multiply by their ratio we will have $\frac{1}{2} = 1 \times \frac{1}{2} = \frac{1}{2}$, or if we multiply in, we will have

$$\frac{1}{2} = \frac{4 \times 1}{4 \times 2} = \frac{4}{8} = \frac{1}{2};$$

but if 4 and 4 be reduced to infinitesimally small quantities and we multiply in we will have

$$\frac{1}{2} = \frac{0}{0} = 1,$$

or if 4 and 4 be made enormously large quantities we will have

$$\frac{1}{2} = \frac{\infty}{\infty} = 1.$$

And hence the symbol $\frac{0}{0}$

tesimals, and $\frac{\infty}{\infty}$ is the ratio of infinitesimals, and $\frac{\infty}{\infty}$ is its reciprocal; and the ratio of these ratios is 1; thus

$$\frac{0}{0} \div \frac{\infty}{\infty} = 1 \text{ and } \frac{\infty}{\infty} \div \frac{0}{0} = 1,$$

i. e., the ratio of ratios, which are reciprocal, is always the denominate unit; and hence the true significance of $\frac{0}{0}$

and $\frac{\infty}{\infty}$ is ratio of reciprocal ratios.

If we take the equation $x = \frac{a^2 - b^2}{(a - b)^2}$

$$6. \quad x = \frac{a^2 - b^2}{(a - b)^2}$$

will have 7. $x = \frac{0}{0}$; and make $a = b$ infinitesimally we

but by factoring and cancelling we will have

$$8. \quad x = \frac{a+b}{a-b} \times \frac{a-b}{a-b} = \frac{a+b}{a-b} = \frac{2a}{0} = \infty.$$

Now if by 0 we mean absolutely nothing then $x = 2a$, and

$x = \frac{2a}{1}$ i. e., $\frac{2a}{1}$ will be the true ratio of x to 1; and if by 0 we mean an infinitesimal quantity then $x = \infty$ and $\frac{x}{1} = \frac{\infty}{1}$, i. e., $\frac{\infty}{1}$ will be the true ratio of x

to 1; but the two values of x are incommensura, i. e., in the first case it is finite and in the second it is infinite: and we will have the proposition

$$9. \quad x:1::\infty:1, \quad x \times 1 = \infty + 1 = \infty. \quad \text{Again take the equation}$$

$$10. \quad x = \frac{(a-b)^2}{a^3 - b^3}$$

and by making $a = b$ infinitesimally we will

have 11. $x = \frac{0}{0}$, this last equation may be stated thus $\frac{1}{3a^2}$

and if by 0 we mean absolutely nothing we will have $\frac{1}{3a^2}$, i. e., $\frac{1}{3a^2}$ will be

the true ratio of 1 to x , and if by 0 we mean an infinitesimal then $x = 0$ and

we will have $\frac{1}{x} = \frac{1}{0}$; but the two values of x are incommensura, i. e., in the

first case it is finite and in the second it is infinitesimal: and we will have the proportion $x:1::0:1$, $x \times 1 = 0 \times 1 = 0$. And from the above we see

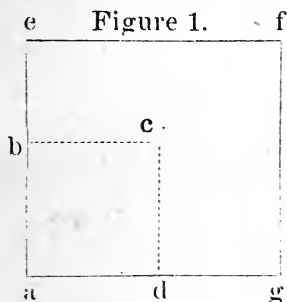
that $\frac{0}{1}$, or $\frac{1}{0}$, or $\frac{\infty}{1}$, or $\frac{1}{\infty}$, may be a ratio and may have a ratio. And we

may have $\frac{\infty}{0}$, and $\frac{0}{\infty}$; and hence $\frac{\infty}{0}$ or $\frac{0}{\infty}$ may be a ratio and may

have a ratio, and they and their ratios are the reciprocals of each other.

Now the whole object of differential calculus is to determine the ratio of rates, i. e., to determine the ratio of ratios; for rate and ratio, when applied to motion or increase, are the same thing. And the ratio of one constant

number to another is easily found by the ordinary principles of Arithmetic; it is easy also to find the ratio of rates of the movements of two bodies, when their rates are uniform, i. e., when each one for itself makes commensural spaces in commensural times; but when the rate of one is uniform and the rate of the other proceeds upon some law other than that of uniformity, i. e. when it does not make commensural spaces in commensural times, a case is presented for the differential calculus. Let us then examine the following Theorem in the calculus: "The rate of variation of the side of a square is to that of its area, in the ratio of unity to twice the side of the square." This is the enunciation of the Theorem as given by Prof. Loomis; as we consider, however, that this enunciation is incorrect and does not set out clearly the matter to be proven, we will give the following in its stead: The rate of variation of the side of a square is to the rate of variation of the corresponding area, in the ratio of unity to twice the side of the unvaried square



+ the variation of the side. Let a.b. (Fig. 1.) be the side of the square a, b, c, d and a, and suppose this side to be elongated to e in one second of time, b e will then be its increase and the corresponding increase of area will be the space b, e, f, g, d, c, b: let $h = b e$, and $g = b e f g d c b$, then $h =$ increase of the side, and $g =$ the corresponding increase of area. Now as $h =$ the increase of the side in one second, the rate of this increase will

$$\frac{h}{1}$$

and as $g =$ the corresponding increase of area, its rate of increase will

$$be = \frac{g}{1}; \text{ and the ratio of these rates will be } \frac{h}{1} \div \frac{g}{1} = \frac{h}{g}.$$

Now let $x = ab$ —the side of the square abcd, and $y = ae$ —the side of the square aefga; then $y - x = h$, and $y^2 - x^2 = g$, and consequently,

$$10. \quad \frac{h}{g} = \frac{y-x}{y^2-x^2} = \frac{y-x}{y-x} \times \frac{1}{y+x} = \frac{1}{y+x}.$$

But $y + x = 2x + h$, and therefore:

$$11. \quad \frac{h}{g} = \frac{1}{2x+h}.$$

But as the value of g depends upon the value of h , if we make h an infinitesimal (and we have seen in 2§ that the ratio of infinitesimals is the same as the ratio of appreciable quantities springing from them by multiplication) we will have:

$$12. \quad \frac{0}{0} = \frac{1}{2x+0}.$$

and hence for infinitesimal variations we have:

$$h:g::1:2x.$$

NOTE.—To treat specially of mathematics is not our object in this work, nor do we wish by criticising to offer refutations: but as the under-

standing of ratio is important and as the calculus treats specially of this subject, to set it upon clear and true foundations must be acceptable to every student. And from the above demonstration it will appear to every reflecting reader, that the ideas entertained by many teachers of the calculus, that $h \div g$ is not the true ratio of the rate of increase of the side of a square to the rate of increase of the corresponding area, but that in order to get at the true ratio we must reduce h and g to infinitesimal quantities, so that their difference may be less than any assignable quantity, supposing that thereby the ratio will be the true ratio to within less than any assignable quantity, is erroneous.

Again, take the Theorem: The rate of variation of the edge of a cube is to the rate of variation of the corresponding solidity, in the ratio of unity to the square of the varied edge + the product of the varied and unvaried edges + the square of the unvaried edge. Let h = the variation of edge, and g = corresponding variation of solidity; and let y = edge of varied cube, and x = edge of unvaried cube; then

$$13. \quad \frac{h}{g} = \frac{y-x}{y^3-x^3} = \frac{1}{y^2+yx+x^2}.$$

If within this equation $y=x$ to within less than any assignable difference, h and g will become infinitesimals and we will have

$$14. \quad \frac{0}{0} = \frac{1}{3x^2}:$$

and hence for infinitesimal variations, $h:g::1:3x^2$. If the edge be decreasing instead of increasing $x > y$ and we will have

$$15. \quad \frac{-h}{-g} = \frac{x-y}{x^3-y^3}, \text{ and when } x=y \quad \frac{-h}{-g} = \frac{1}{3x^2}.$$

When the motion or variation of one body or thing is uniform and another body or thing makes commensural increments of increase or decrease of variation in consecutive commensural times, the latter body or thing varies in Arithmetrical progression; and in order to get the ratio of the ratios of the variations we must divide the ratio of space made by the first object in a given time by the ratio of the space made by the second object in a commensural time. Let h = space made in five minutes by an object making uniformly b feet per minute, and let another object move, making a feet for the first minute $a+b$ for the second, $a+2d$ for the third and so on for five minutes with the commensural increment of increase of d in each successive minute:

then $\frac{h}{5}$ = ratio of the first objects variation, and letting S stand for the sum of the terms in the second objects variation, $\frac{S}{5}$ = ratio of second object's variation and $\frac{h}{S}$ = ratio of these ratios. But letting n = number of terms, and l = last term, and h will be equal to bn , and $S = \left[\frac{a+b}{2} \right] n$, and hence:

$$15. \quad \frac{h}{S} = \frac{bn}{\left[\frac{a+1}{2} \right]^n} = \frac{2b}{a+1}. \quad \text{But } l = a + (n-1)d, \text{ and hence;}$$

$$16. \quad \frac{h}{S} = \frac{2b}{2a + (n-1)d}, \text{ and when } n=1$$

$$17. \quad \frac{h}{S} = \frac{b}{a}, \text{ and if we reduce } h \text{ and } S \text{ to infinitesimals,}$$

$$18. \quad \frac{0}{0} = \frac{b}{a} : \frac{b}{a}, \text{ therefore, is the true ratio of the objects' variations}$$

at the infinitesimal point from which they begin to vary.

The equation $\frac{a}{1} = \frac{a}{a + (n-1)d}$ gives the ratio of the first term in an Arithmetrical progression to the last term considered. If the reader does not fully comprehend this and the following paragraphs, let him turn to some mathematical work upon the subjects.

If one object vary uniformly and another object vary in such manner that the successive values made in commensural times are in proportion to each other, i. e., the terms have a constant ratio, the latter object's variations are in Geometrical progression; and we find the ratio of these objects' variations by dividing the ratio of the one by the ratio of the other. Using the letters as in the preceding paragraph with the addition of r for the constant ratio, and relying upon the readers knowledge of mathematics we will have:

$$20. \quad \frac{h}{S} = \frac{bn}{ar^n - a} = \frac{bn(r-1)}{a(r^n - 1)}. \quad \text{But when } n=1 \text{ we will have}$$

$$21. \quad \frac{h}{S} = \frac{b}{a}, \text{ and consequently } \frac{b}{a} \text{ will be the true ratio of these}$$

objects' variations at the zero point of varying. The equation

$$22. \quad \frac{a}{1} = \frac{a}{ar^n - 1} \text{ gives the ratio of the first term to the last term considered.}$$

If an object vary in Arithmetrical progression and another by Geometrical progression and we use capital letters in the Geometrical equation for the sum and first term we will have

$$23. \quad \frac{s}{S} = \frac{\left[\frac{a+1}{2} \right]^n}{\frac{1r - A}{r-1}} = \frac{[r(a+1) - (a+b)]^n}{2(1r - A)}.$$

We have gone far enough, perhaps, upon the subject of ratio.

CHAPTER XI.

TRANSFORMATION OF PROPOSITIONS.

If we take the three distinct propositions:

$$1. \quad \frac{\frac{\wedge \wedge}{TS}}{a} \quad \frac{\frac{\wedge \vee}{TS}}{<} \quad \frac{\frac{\wedge \wedge}{TS}}{b}, \quad \frac{\frac{\wedge \wedge}{TS}}{c} \quad \frac{\frac{\wedge \vee}{TS}}{<} \quad \frac{\frac{\wedge \wedge}{TS}}{d}, \quad \frac{\frac{\wedge \wedge}{TS}}{a} \quad \frac{\frac{\wedge \vee}{TS}}{=} \quad \frac{\frac{\wedge \wedge}{TS}}{d},$$

by uniting them into one we may have

$$2. \quad \frac{\frac{\wedge \vee}{TS}}{a} \quad \frac{\frac{\wedge \vee}{TS}}{< <} \quad \frac{\frac{\wedge \vee}{TS}}{b}$$

And if in proposition 2. we place the sign \wedge or \vee by the side of the terms, not as signs of homon or hetero, but simply as the sign of incommensura we will have

$$3. \quad \frac{\frac{\wedge \vee}{TS}}{V | a} \quad \frac{\frac{\wedge \vee}{TS}}{< <} \quad \frac{\frac{\wedge \vee}{TS}}{b | V}$$

If we take the distinct propositions

$$4. \quad \frac{\frac{\wedge \wedge}{TS}}{a} \quad \frac{\frac{\wedge \vee}{TS}}{<} \quad \frac{\frac{\wedge \wedge}{TS}}{b}, \quad \frac{\frac{\wedge \wedge}{TS}}{b} \quad \frac{\frac{\wedge \vee}{TS}}{=} \quad \frac{\frac{\wedge \wedge}{TS}}{c}, \quad \frac{\frac{\wedge \wedge}{TS}}{a} \quad \frac{\frac{\wedge \vee}{TS}}{=} \quad \frac{\frac{\wedge \wedge}{TS}}{d},$$

by uniting them into one we may have

$$5. \quad \frac{\frac{\wedge \vee}{TS}}{\wedge | a} \quad \frac{\frac{\wedge \vee}{TS}}{=<} \quad \frac{\frac{\wedge \vee}{TS}}{b | \vee}$$

And from proposition 5 by writing the sign of incommensura under the terms we may have the propositions

$$6. \quad \frac{\frac{\wedge \vee}{TS}}{a+c} \quad \frac{\frac{\wedge \vee}{TS}}{=<} \quad \frac{\frac{\wedge \vee}{TS}}{b+d}, \quad 7. \quad \frac{\frac{\wedge \vee}{TS}}{a \times c} \quad \frac{\frac{\wedge \vee}{TS}}{=<} \quad \frac{\frac{\wedge \vee}{TS}}{b \times d}, \quad 8. \quad \frac{\frac{\wedge \vee}{TS}}{\wedge | a} \quad \frac{\frac{\wedge \vee}{TS}}{=<} \quad \frac{\frac{\wedge \vee}{TS}}{b | \vee}$$

If we take the commensural propositions

$$9. \quad \frac{\frac{\wedge \wedge}{TS}}{a} \quad \frac{\frac{\wedge \vee}{TS}}{=} \quad \frac{\frac{\wedge \wedge}{TS}}{b}, \quad \frac{\frac{\wedge \wedge}{TS}}{c} \quad \frac{\frac{\wedge \vee}{TS}}{=} \quad \frac{\frac{\wedge \wedge}{TS}}{d}, \quad \frac{\frac{\wedge \wedge}{TS}}{a} \quad \frac{\frac{\wedge \vee}{TS}}{=} \quad \frac{\frac{\wedge \wedge}{TS}}{d},$$

by using the sign of commensura (not that of similia) by the side of the

terms we may have

$$10. \frac{\hat{T} \overset{\vee}{S}}{\parallel \left| \begin{array}{c} a \\ c \end{array} \right.} = \frac{\hat{T} \overset{\vee}{S}}{\parallel \left| \begin{array}{c} b \\ d \end{array} \right.} = \frac{\hat{T} \overset{\vee}{S}}{\parallel \left| \begin{array}{c} b \\ d \end{array} \right.} \parallel, \text{ from which we may have } 11. \frac{\hat{T} \overset{\vee}{S}}{\frac{a+c}{=}} = \frac{\hat{T} \overset{\vee}{S}}{\frac{b+d}{=}} = \frac{\hat{T} \overset{\vee}{S}}{=}$$

$$12. \frac{\hat{T} \overset{\vee}{S}}{\frac{a \times c}{=}} = \frac{\hat{T} \overset{\vee}{S}}{\frac{b \times d}{=}} = \frac{\hat{T} \overset{\vee}{S}}{\frac{b \times d}{=}} \text{ and } 13. \frac{\hat{T} \overset{\vee}{S}}{\parallel \left| \begin{array}{c} a \\ c \end{array} \right.} = \frac{\hat{T} \overset{\vee}{S}}{\parallel \left| \begin{array}{c} b \\ d \end{array} \right.} \parallel$$

If we have any number of incommensural propositions as the following:

$$14. \frac{\hat{T} \hat{S}}{a} < \frac{\hat{T} \overset{\vee}{S}}{b}, \frac{\hat{T} \hat{S}}{b} < \frac{\hat{T} \overset{\vee}{S}}{c}, \frac{\hat{T} \hat{S}}{c} < \frac{\hat{T} \overset{\vee}{S}}{d}, \text{ etc.}$$

we may derive from them

$$15. \frac{\hat{T} \overset{\vee}{S}}{\frac{a+d}{<}} = \frac{\hat{T} \hat{S}}{\wedge <} = \frac{\hat{T} \overset{\vee}{S}}{\frac{a+d}{<}} \quad 16. \frac{\hat{T} \overset{\vee}{S}}{\frac{a \times d}{<}} = \frac{\hat{T} \hat{S}}{\wedge <} = \frac{\hat{T} \overset{\vee}{S}}{\frac{a \times d}{<}} \quad 17. \frac{\hat{T} \overset{\vee}{S}}{\wedge \left| \begin{array}{c} a \\ d \end{array} \right.} = \frac{\hat{T} \hat{S}}{\wedge <} = \frac{\hat{T} \overset{\vee}{S}}{\frac{a}{d} \left| \wedge \right.}, \text{ etc.}$$

And if we have any number of incommensural propositions as the following:

$$18. \frac{\hat{T} \hat{S}}{a} < \frac{\hat{T} \overset{\vee}{S}}{b}, \frac{\hat{T} \hat{S}}{b} < \frac{\hat{T} \overset{\vee}{S}}{c}, \frac{\hat{T} \hat{S}}{c} < \frac{\hat{T} \overset{\vee}{S}}{d}, \frac{\hat{T} \hat{S}}{d} < \frac{\hat{T} \overset{\vee}{S}}{e}, \frac{\hat{T} \hat{S}}{e} < \frac{\hat{T} \overset{\vee}{S}}{f}, \text{ etc.}$$

we may derive from them

$$19. \frac{\hat{T} \overset{\vee}{S}}{\frac{a+b+c+d+e+f}{<}} = \frac{\hat{T} \hat{S}}{\wedge <} = \frac{\hat{T} \overset{\vee}{S}}{\frac{a+b+c+d+e+f}{<}}, \text{ etc.}$$

By setting down all the signs in our transformations, we are able to integrate or resolve the complex propositions into their simple and primitive ones without any difficulty; but there is still another object of more importance in doing so, as we will see hereafter.

Now we have shown heretofore, that both incommensural and commensural propositions contain only relations inter se similia, and as we have used the letters a, b, c, etc., not to distinguish kinds of things, but merely to distinguish the quantities of similia, proposition 6 may be transformed into

$$20. \quad \frac{\frac{\wedge \vee}{TS}}{a+c} \quad \frac{\frac{\wedge \wedge}{TS}}{||} \quad \frac{\frac{\wedge \vee}{TS}}{b+d}.$$

$$< \qquad \qquad \qquad >$$

And any commensural or incommensural, or commensural incommensural, or incommensural commensural proposition may be transformed by striking out the signs of equality and inequality between the terms and inserting in their stead the sign of similia.

Now let the letters a, b, c, etc., stand for names, which distinguish similia and differentia and take the differential propositions

$$21. \quad \frac{\frac{\wedge \wedge}{TS}}{a} \quad \frac{\frac{\wedge \vee}{TS}}{+} \quad \frac{\frac{\wedge \wedge}{TS}}{b} \quad \text{and} \quad \frac{\frac{\wedge \wedge}{TS}}{c} \quad \frac{\frac{\wedge \vee}{TS}}{+} \quad \frac{\frac{\wedge \wedge}{TS}}{d}. \quad \text{By uniting them we will have}$$

$$22. \quad \frac{\frac{\wedge \vee}{TS}}{a} \quad \frac{\frac{\wedge \vee}{TS}}{++} \quad \frac{\frac{\wedge \vee}{TS}}{b}$$

$$c \qquad \qquad \qquad d$$

Take the propositions

$$23. \quad \frac{\frac{\wedge \wedge}{TS}}{a} \quad \frac{\frac{\wedge \vee}{TS}}{+} \quad \frac{\frac{\wedge \vee}{TS}}{b} \quad \frac{\frac{\wedge \wedge}{TS}}{a'} \quad \frac{\frac{\wedge \vee}{TS}}{+} \quad \frac{\frac{\wedge \wedge}{TS}}{b'}. \quad \text{By uniting we will have}$$

$$24. \quad \frac{\frac{\wedge \vee}{TS}}{a} \quad \frac{\frac{\wedge \vee}{TS}}{+} \quad \frac{\frac{\wedge \vee}{TS}}{b} \quad \text{or,} \quad 25. \quad \frac{\frac{\wedge \vee}{TS}}{a} \quad \frac{\frac{\wedge \vee}{TS}}{+} \quad \frac{\frac{\wedge \vee}{TS}}{b}$$

$$a' \qquad \qquad \qquad b' \qquad \qquad \qquad b'$$

As the letters a, b, c, etc., are distinguishing names we need not place any sign by the side of the terms, as we can integrate without doing so.

Take the propositions

$$26. \quad \frac{\frac{\wedge \wedge}{TS}}{a} \quad \frac{\frac{\wedge \vee}{TS}}{||} \quad \frac{\frac{\wedge \wedge}{TS}}{a'} \quad \frac{\frac{\wedge \wedge}{TS}}{a''} \quad \frac{\frac{\wedge \vee}{TS}}{||} \quad \frac{\frac{\wedge \wedge}{TS}}{a'''}. \quad \text{By uniting them we have}$$

$$27. \quad \frac{\frac{\wedge \vee}{TS}}{a} \quad \frac{\frac{\wedge \vee}{TS}}{||} \quad \frac{\frac{\wedge \vee}{TS}}{a'}$$

$$a'' \qquad \qquad \qquad a'''$$

Now we have heretofore shown that the mind's capacity to heterate depends upon time and space; and it may, perhaps, be well enough to make a single remark further on that subject here. If a bell be struck, we can both see the bell and hear its sound in an homonical time, while the bell occupies an homonical space: but the organs of vision and those of hearing occupy heterical spaces; and the sound and the light coming from the bell do not come to the mind through homonical spaces, i. e., although there be

apparently in the case homonical time and spaces yet the spaces are really hetera, and they enable the mind to heterate. Take the heterical propositions

$$28. \quad \frac{\hat{T}\hat{S}}{a} \quad \frac{\hat{T}\overset{\vee}{S}}{V} \quad \frac{\hat{T}\hat{S}}{a'} \quad \frac{\hat{T}\hat{S}}{b} \quad \frac{\hat{T}\hat{S}}{\Lambda} \quad \frac{\hat{T}\hat{S}}{b'}. \quad \text{By uniting them we will have}$$

$$29. \quad \frac{\hat{T}\overset{\vee}{S}}{a} \quad \frac{\hat{T}\overset{\vee}{S}}{VV} \quad \frac{\hat{T}\overset{\vee}{S}}{a'}. \\ \frac{\quad}{b} \quad \frac{\quad}{b'}$$

But if we should transpose the terms of the first of propositions 28 and then unite it with itself we would have

$$30. \quad \frac{\hat{T}\overset{\vee}{S}}{a} \quad \frac{\hat{T}\hat{S}}{\Lambda V} \quad \frac{\hat{T}\overset{\vee}{S}}{a'}. \\ \frac{\quad}{a'}$$

Take the propositions

$$31. \quad \frac{\hat{T}\hat{S}}{a} \quad \frac{\hat{T}\hat{S}}{\Lambda} \quad \frac{\hat{T}\hat{S}}{a} \quad \frac{\hat{T}\hat{S}}{b} \quad \frac{\hat{T}\hat{S}}{\Lambda} \quad \frac{\hat{T}\hat{S}}{b}. \quad \text{And by uniting them we have}$$

$$32. \quad \frac{\hat{T}\hat{S}}{a} \quad \frac{\hat{T}\hat{S}}{\Lambda V} \quad \frac{\hat{T}\hat{S}}{a}. \\ \frac{\quad}{b} \quad \frac{\quad}{b}$$

But if we unite the first of the propositions 31 with itself we will have

$$33. \quad \frac{\hat{T}\hat{S}}{a} \quad \frac{\hat{T}\hat{S}}{\Lambda \Lambda} \quad \frac{\hat{T}\hat{S}}{a}. \\ \frac{\quad}{a} \quad \frac{\quad}{a}$$

Now from the few examples given above any one with moderate capacity can see how to unite and transform simple propositions into complex ones and obtain all the varieties of propositions having the varieties of signs between the terms as set down in Chapter First of this book and to place the appropriate signs over the T's and S's, we need not therefore deal further with this matter.

Now we have seen in Book I, that in every case of causation some homon is converted into hetera or vice versa; some similia are converted into differentia or vice versa; or, some commensura are converted into incommensura or vice versa: and if we compare the simple propositions with the complex ones derived from them in the preceeding transformations, on comparison of the signs of the S's over the terms we will see, that in the transformations given the heteration of space has occurred. In those transformations of propositions, however, the heteration of space may have been made merely by the mind; but if we suppose a, b, c, etc., to be material

objects and to have changed the relations in which they existed as expressed in the simple propositions, into the relations as expressed in the derivative propositions, then causes external to the mind bringing about these changes have involved the heteration of space. In the transformation of simple homonical propositions into homonical homonical propositions, indeed, no such change in the signs of the S's is indicated, nor could the heteration of space occur, were A, for instance, a material object and contemplated in its different mental relations.

But in Book II we saw that among causes a homon of time and a homon of space are the necessary conditions of causation, and also that effects spring from heterical causes. Let us suppose, therefore, a, b, c, etc., in the foregoing complex propositions to be causes, and let us make a homon of time and a homon of space over the terms and see what changes follow. If in proposition 6, we change V into \wedge over the S's on the terms, we can not write the new proposition resulting without performing the addition; but letting v stand for the sum, we may then write the new proposition and have

$$34. \quad \frac{\wedge\wedge}{TS} \quad \frac{\wedge^V}{TS} \quad \frac{\wedge\wedge}{v'}$$

We may deal with propositions 7 and 8 in a similar manner.

If we change V into \wedge over the terms of proposition 11 and let v stand for the sum, we will have

$$35. \quad \frac{\wedge\wedge}{TS} \quad \frac{\wedge^V}{TS} \quad \frac{\wedge\wedge}{v'}$$

And if in this proposition we change V into \wedge between the terms we will have

$$36. \quad \frac{\wedge\wedge}{TS} \quad \frac{\wedge\wedge}{\wedge} \quad \frac{\wedge\wedge}{v}$$

From which changes we see, that the homonating of the spaces between the objects in the terms produces effects inter se heter., but each of which per se is a homon; and the homonating of the spaces between the terms produces an effect per se homon: the converse is also true.

If in proposition 22 we change V into \wedge over the terms two effects must be produced; and as ac and bd are differential differentia the effects inter se must be differentia. Let x stand for one of the effects and y for the other, and we will have

$$37. \quad \frac{\wedge\wedge}{TS} \quad \frac{\wedge^V}{TS} \quad \frac{\wedge\wedge}{y}$$

And if we change V into \wedge between the terms we will have an effect differing from both x and y, i. e., we will have

$$38. \quad \frac{\frac{\wedge \wedge}{TS}}{z} \quad \frac{\frac{\wedge \wedge}{TS}}{\wedge} \quad \frac{\frac{\wedge \wedge}{TS}}{z}.$$

Let us now take propositions 23 and go through all the transformations, which the reader will now readily understand

$\frac{\wedge \wedge}{TS}$	$\frac{\wedge \vee}{TS}$	$\frac{\wedge \wedge}{TS}$
a	\vdash	b
$\frac{\wedge \wedge}{TS}$	$\frac{\wedge \vee}{TS}$	$\frac{\wedge \wedge}{TS}$
a'	\vdash	b'

\mathcal{U}

Produce by heterating space between objects in terms towards each other.

$\frac{\wedge \vee}{TS}$	$\frac{\wedge \vee}{TS}$	$\frac{\wedge \vee}{TS}$
a	$\parallel \vdash$	a'
b'		b

\mathcal{U}

By homonating space between objects in terms.

$\frac{\wedge \wedge}{TS}$	$\frac{\wedge \vee}{TS}$	$\frac{\wedge \wedge}{TS}$
x	\parallel	x

\mathcal{U}

By homonating space between terms.

$\frac{\wedge \wedge}{TS}$	$\frac{\wedge \wedge}{TS}$	$\frac{\wedge \wedge}{TS}$
x	\wedge	x

\mathcal{U}

By heterating space between terms.

$\frac{\wedge \wedge}{TS}$	$\frac{\wedge \vee}{TS}$	$\frac{\wedge \wedge}{TS}$
x	\parallel	x

\mathcal{U}

By heterating space between objects of terms.

$\frac{\wedge \vee}{TS}$	$\frac{\wedge \vee}{TS}$	$\frac{\wedge \vee}{TS}$
a	$\parallel \vdash$	a'
b'		b

2f

By heterating space between objects of terms from each other.

$\frac{\wedge \wedge}{TS}$	$\frac{\wedge \vee}{TS}$	$\frac{\wedge \wedge}{TS}$
a	H	b
$\frac{\wedge \wedge}{TS}$	$\frac{\wedge \vee}{TS}$	$\frac{\wedge \wedge}{TS}$
a'	H	b'

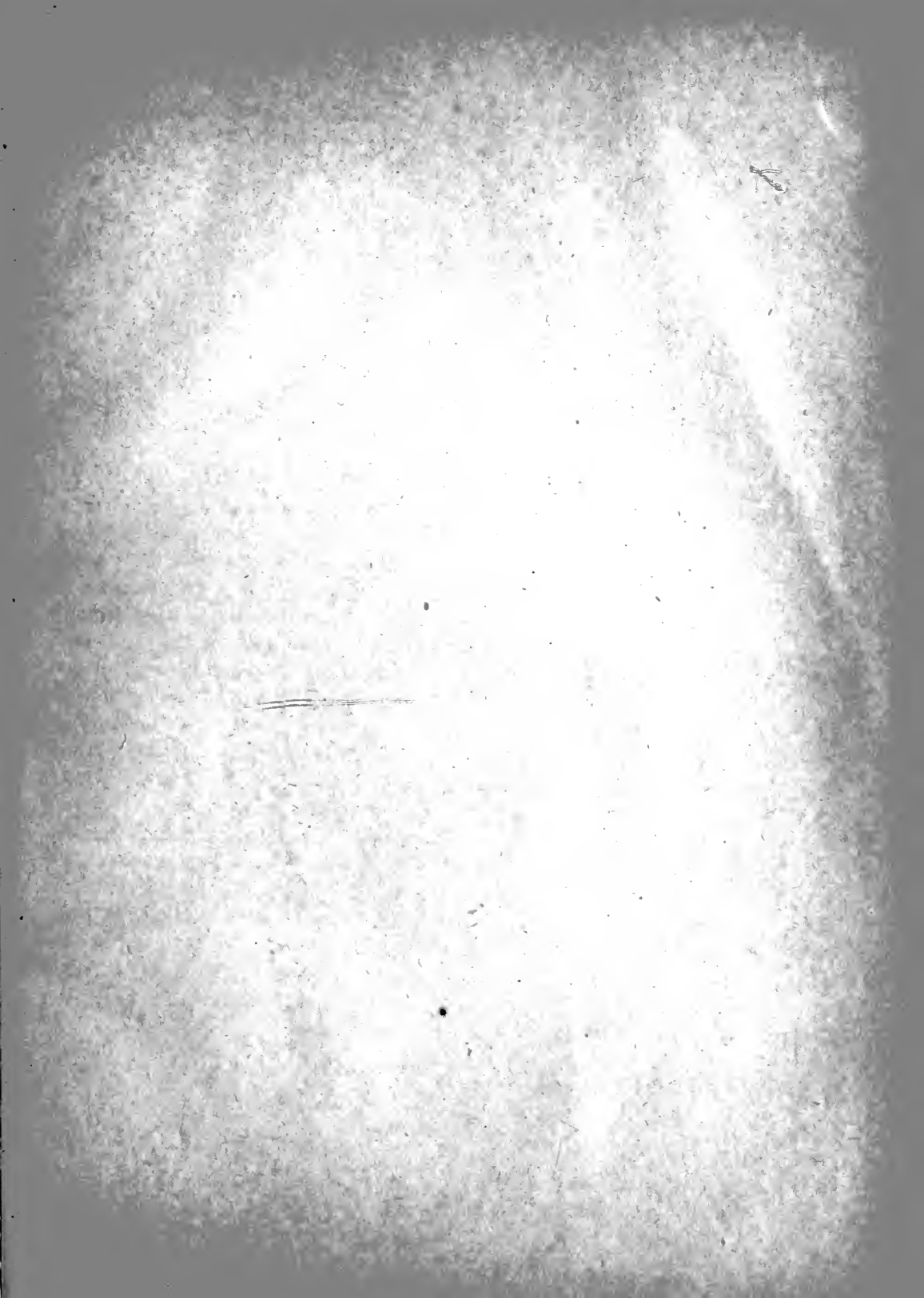
We have now gone far enough upon the subject of Transformations of Propositions to give the reader a thorough understanding of the matter, if he will study and use his own mind in working out upon a slate the various transformations possible, in order to familiarize the modes of reasoning.

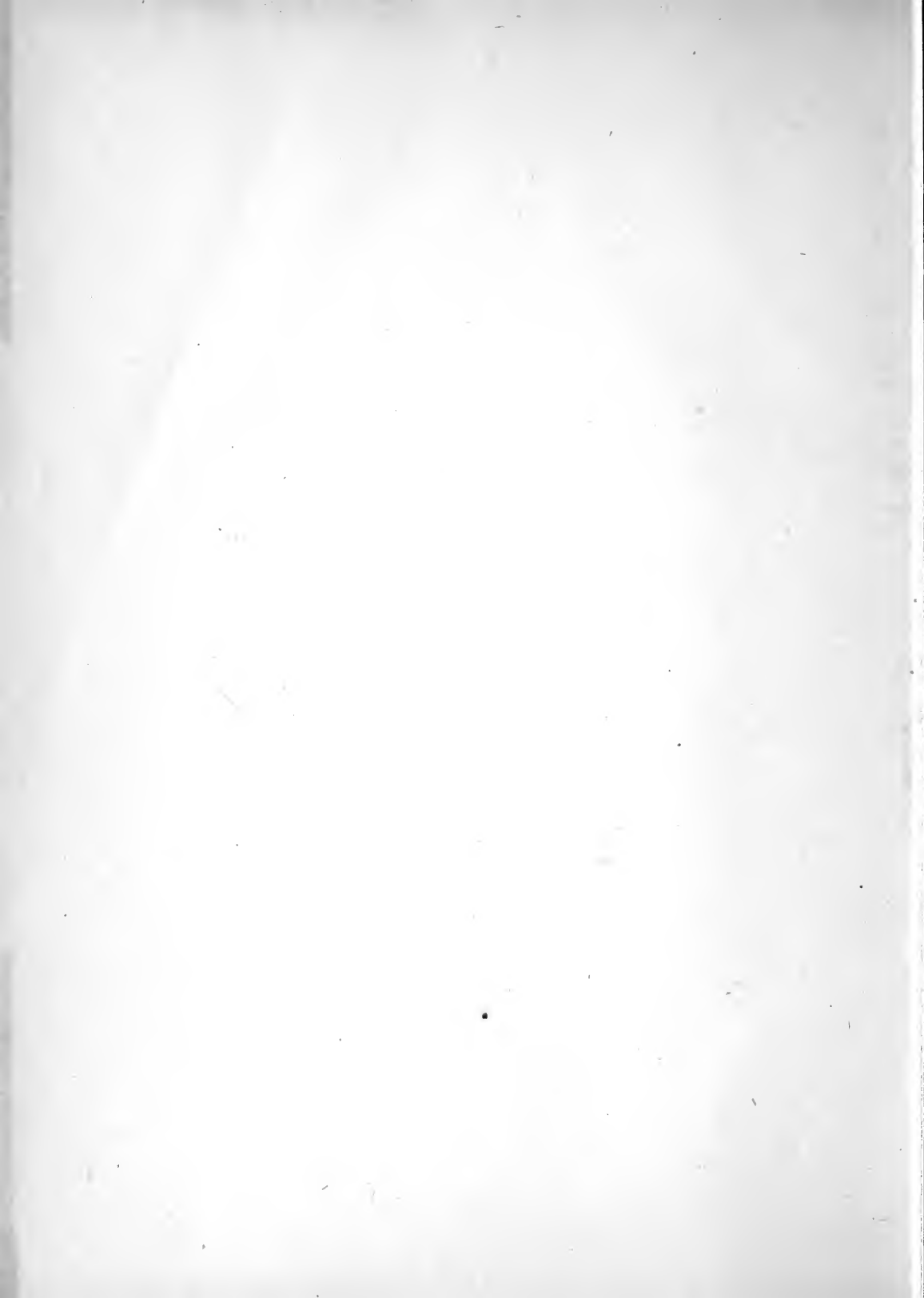
CONCLUDING REMARKS.

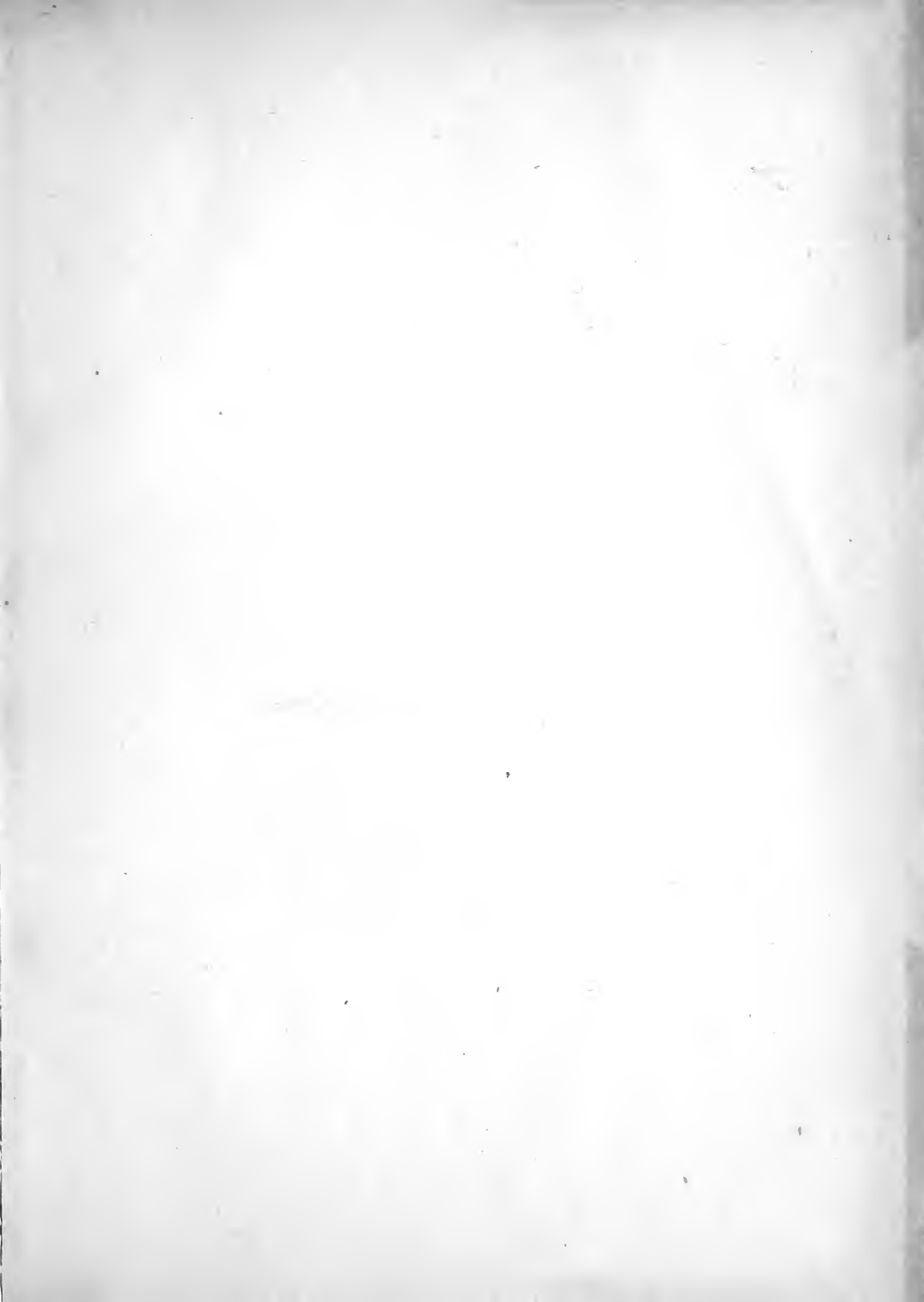
It was the intention of the author to have continued this book much further than its present limits, to treat of the ratio of gregaria and of their combinations in propositions by which what are called the laws of nature can be evolved, to point out errors in the fundamental principles in natural philosophy, to state experiments made and demonstrable results actually obtained in light, electricity and heat. But hard times, ill health, and the great difficulty in getting the authors ideas in print at all, so as to place them before the scientific world, have compelled the him to stop here; although the subject is abruptly broken off and much to his regret the applicability of the science to the investigation of nature is not exhibited, the author claims that he has made many valuable discoveries in physical science which must be left for another work and for more auspicious circumstances, if such should ever come. The present edition has been put in print under the most harrising circumstances and difficulties, and it cannot be expected to be otherwise than that numerous errors and obscurities should appear in it. These the reader will excuse, and when scientific men shall have investigated the work and expressed their opinion about it the author will be better prepared to judge of the expediency of making the attempt to complete a work on natural philosophy based upon the principles of experiment and reasoning exhibited in this book.

THE END.

E. H. Burn











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